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An Overview of Probabilistic Performance Analysis Methods for Large Scale and Time-Dependent Systems

Various algorithms have been proposed to evaluate the performance of large scale and time-dependent systems. Probabilistic approaches based on computer experiments have been shown to be more efficient than classical approaches such as Monte-Carlo simulations for a various number of practical cases. This article thus gives an overview of the latest trends in the field, applied to the performance evaluation of large scale or time-dependent system. Different realistic applications in the aerospace domain are considered.

Introduction

In the aerospace and defense domain, realistic systems such as defense architecture or air traffic management are very time consuming and difficult to simulate with accuracy. Evaluating system behavior in order to improve performance or to evaluate alternative system design strategies thus requires system performances to be predicted for a given set of inputs or parameter values. To remain computationally feasible, this prediction requires a specific computational physics model, performance evaluation methods and uncertainty management.

In the context of model uncertainties, simulation tools are required to directly include uncertainty in system components and architecture, providing a set or a distribution of possible outcomes, rather than a single one. Uncertainty is considered as a measure of how large the deviations of a process from a predicted behavior can be. The classical approach consists in directly simulating the model using Monte-Carlo methods. A large number of samples of the parameter vector subject to uncertainty are randomly chosen, yielding a correspondingly large number of simulations, which may result in intractable complexity. Various alternatives have been developed to drastically improve crude Monte-Carlo performances.

This paper presents a review of a wide range of probabilistic methods that have been developed to estimate complex system performances. In the first section, Monte-Carlo methods are considered, since they can be applied in various frameworks. The last two sections describe alternative techniques that overcome some of the limitations of the Monte-Carlo approach.

Monte-Carlo approach

There are several approaches for taking into account parametric uncertainties within a system evaluation: interval analysis, fuzzy sets, fuzzy logic or possibility theory [1][29][107][22]. However, the probabilistic approach is the one that offers many powerful theoretical and numerical tools. The Monte-Carlo approach [83][85] is based on modeling each uncertain parameter through a random variable. The system then becomes a random parameter system and the output itself becomes a random variable $X(\omega)$, which must be characterized through its probability distribution $p_X(\omega)$. This may prove to be a very difficult task since there is almost never an analytical expression of the output distribution. The outcomes are usually statistical quantities, expressed as the mean of some output function $E\left[f(X(\omega))\right]$ derived from integrals again involving the probability distribution:

$$E\left[f(X(\omega))\right] = \int f(x)\, p_X(x)\, dx$$

The Monte-Carlo approach provides an estimation of this deterministic integral, by using repeated random samples: one does not need to know the output probability distribution as long as samples are available. This method is based on the Law of Large Numbers.

A natural and important question that arises is related to the convergence rate of this method. The use of the central limit theorem shows that the convergence rate is of order $\sqrt{n}$, where n is the number of samples. However, the convergence rate is independent of the size of the random variable, rendering this approach very efficient for high dimensional problems involving a large number of random $X(\omega)$ parameters. Another nice feature is that no smoothness assumption has to be made for the function f . The implementation of the Monte-Carlo approach is straightforward, since it acts outside of any given numerical code or procedure: one does not need to modify complex industrial codes, or even have any knowledge of them, in order to use it. This property is a key feature, which explains the popularity of the approach. The only requirements are, first, to be able to fit the probability distribution to each uncertain parameter introduced in the system and, secondly, to be able to generate random samplings for these distributions.

An alternative, known as Quasi Monte-Carlo methods, is to replace the random samples by low discrepancy sequences of numbers. However, error bounds are rather difficult to calculate and it appears that such methods should be used for a rather small number of uncertain parameters [69].

The effects of structural uncertainties have been studied in the aeroelastic domain for more than 30 years [74]. As an illustration of the standard Monte-Carlo method, let us consider the problem of evaluating the impact of the geometrical design defaults of an airfoil on the pressure distribution. Geometrical defaults are modeled as a Gaussian random deformation field acting along the airfoil. Figure 1 illustrates a particular simulated geometry.

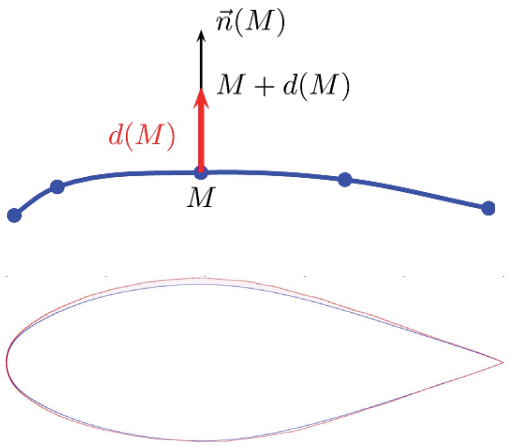


Figure 1 - Geometrical design defaults of an airfoil on the pressure distribution

Figure 2 shows the collection of results obtained by running the Monte-Carlo method with 200 samples of the airfoil geometry. From those results, statistics can be derived, such as the mean value and standard deviation of the shock position, as well as statistics on the stability of the fluid structure coupled system.

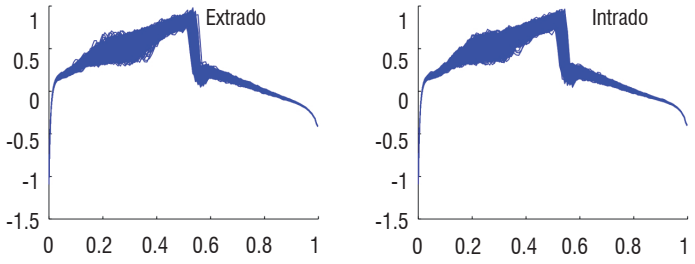


Figure 2 - Monte-Carlo estimation of the shock position

Due to their slow convergence rate, Monte-Carlo methods require a high number of samples. When accurate simulations are very time-consuming, surrogate models for representing the initial system or to assess the dynamic propagation of uncertainty are efficient tools for a limited computing budget. Estimation of very low quantiles may rule out a classical Monte-Carlo approach, since it would require too large a number of simulations

Black box system analysis

When only the resulting outputs are of interest, the black box model can be a crude description of the system. It is characterized by a certain number of input parameters and a few equations that use those inputs to give a set of outputs. This type of model is usually deterministic, so that the same outputs are always obtained for a given set of inputs. When the inputs have random behavior, different issues and questions can be raised. How can the system performance be evaluated if the simulation code is time consuming? What are the most influent inputs of a system on its output values? How can the uncertainty on the inputs be propagated? How can events with low probability (e; g. rare events) be generated?

Meta-modeling

A potential black box representation of a complex system can be supplied by surrogate models, such as response-surface methodology, Kriging, radial basis functions, splines or neural networks [43][93][104][26]. The idea is to substitute the evaluation of some simple function for the costly simulation of the complex system model, in response to a possibly high-dimensional vector of inputs [80]. For example, a prediction of the simulation value over a continuous space can be achieved by combining a space-filling sampling and an accurate interpolation meta-model over the space of interest. It is notably a very efficient strategy when simulation is time consuming. Figure 3 shows the fall-back positions of a space launcher stage, obtained with complex computer code or with a neural network learned from the complex computer code. Circles of Equal Probability (CEP) estimated in both cases are comparable, but the computation time is very low in the case of neural networks. Another application is the search for a global optimum of a criterion, computed via expensive simulation, by sampling only in promising regions of the search space. The surrogate model can then be adjusted accordingly to refine the estimate of this optimum.

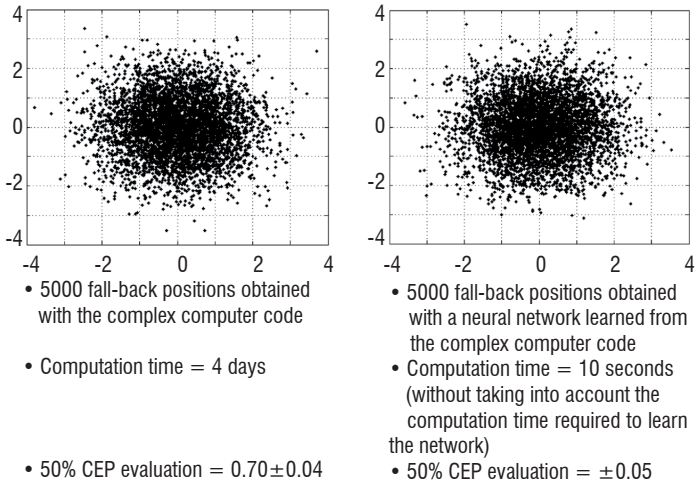


Figure 3 - Fall-back zone estimation of a space launcher stage

Among possible surrogates, Kriging [57][45] models the unknown function as a Gaussian process, which could be seen as the generalization of finite-space Gaussian distributions to a function space of infinite dimension. A correlation function is used to model the influence of the value of the function at one point on its value at another point, depending on a measure of distance between them. This correlation function and its parameters should be chosen according to smoothness assumptions on the function to be approximated. Based on these foundations, Kriging provides a continuous interpolation that is the best linear unbiased prediction of the unknown function. A very interesting property is the possibility to compute the variance of the

prediction error at any predicted point, which can be interpreted as a measure of confidence in the interpolation (see figure 4 , for a simple one-dimensional example). Kriging can also be seen as a linear predictor with a weighted sum on a particular basis of functions, which corresponds to the prior chosen correlation.

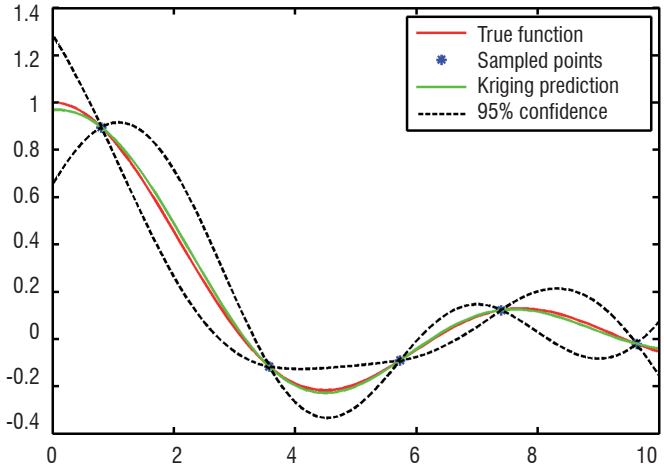


Figure 4 Kriging interpolation with 5 sample points

Moreover, the property of computing the confidence measure has been further exploited to design the so-called efficient global optimization (EGO) algorithm [43]. Its objective is to optimize a criterion, computed via a complex or unknown computer simulation, by iteratively sampling new points in the input search space, where the criterion should be evaluated so as to enhance the estimate of the global optimum.

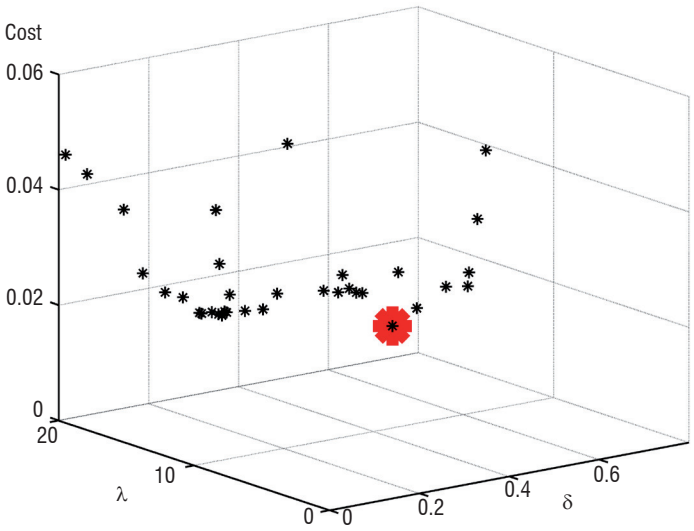


Figure 6 - Tuning of a fault diagnosis method with 2 parameters. The cost represents the trade-off between false-alarm and non-detection rates - exploration of the parameter space by EGO, the estimated optimum is in red

This algorithm has been applied to various design problems, such as multidisciplinary design optimization [92], sensor analysis [62], active recognition [20] (see figure 5) and automatic tuning of fault detection methods [58] (see figure 6). This strategy has provided reliable results with very small samplings for the applications mentioned.

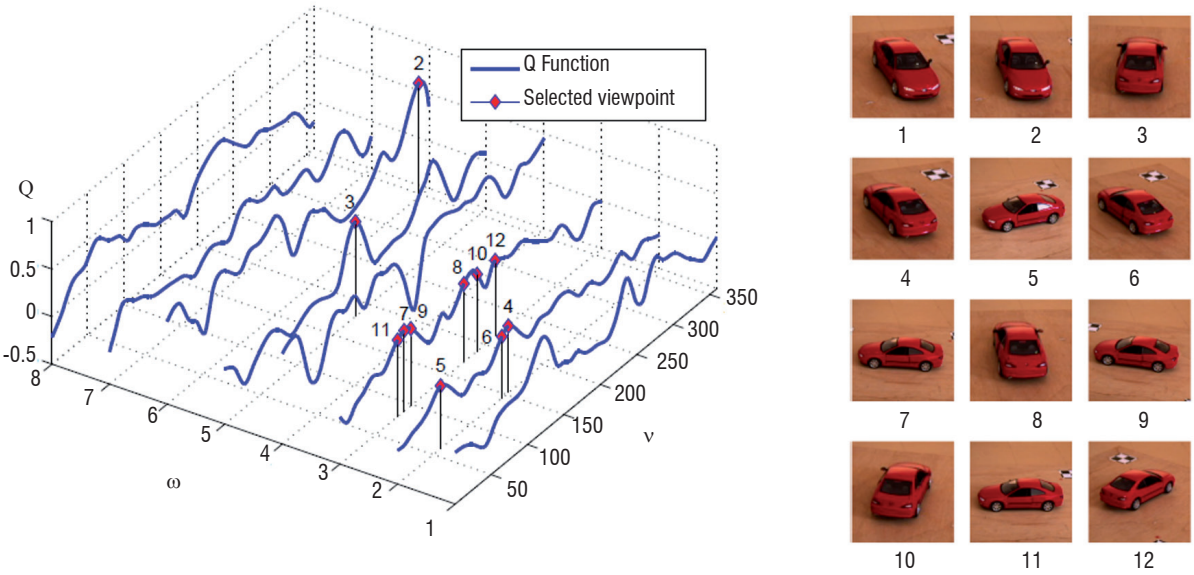


Figure 5 - Active recognition of car type from a database. For each class, the Q-function that determines the angle under which an agent should inspect a vehicle for better recognition has been obtained by Kriging and EGO

Box 1 - Design under uncertainty with Kriging

Available knowledge on uncertainties can be incorporated in different ways into Kriging models, to achieve robust design. In the case of noise on the output of the simulation to be approximated, a Gaussian distribution can be taken into account through the covariance that is at the basis of the Kriging prediction [81]. The resulting linear prediction incorporates the amount of noise to build a smooth fitting function and is thus no longer an exact interpolation of the data. This makes it possible to work with a filtered model from noisy observations, at a low computational cost [100].

For optimization purposes, a generic model is to consider that the computer model under study depends on two types of inputs: control variables and environmental variables [88]. The control variables are those on which optimization is carried out, while the environmental variables are uncontrollable disturbances that affect performance. For instance, a control law or an estimation filter for an aeronautical vehicle are subject to measurement noise, strong uncertainty on the model parameters, variations of the atmospheric conditions (temperature, pressure) and wind turbulence. In such a case, a design of the control variables that is robust to the effect of these environmental variables is sought. When dealing with robust design, a probabilistic or deterministic point of view can be adopted [4]. In the probabilistic framework, a distribution of the environmental variables is assumed and performance is assessed by the expected value of some robustness measure. However, a design that is good on average may prove to be poor for particular values of the environmental variables. In the deterministic framework, it is assumed that the environmental variables belong to some known compact set and performance is assessed by the worst possible value of some robustness measure. The design that is best in the worst case is obviously conservative on average, and the choice between the probabilistic and deterministic points of view should be made on a case-by-case basis. In the probabilistic context, some papers have addressed robust Kriging-based optimization with respect to environmental variables.

In [11][53], Monte-Carlo simulations are performed for each sampled value of a space-filling design of the control variables and a Kriging model is fitted on the resulting mean and variance, before achieving optimization by classical algorithms. In [54], the EGO algorithm has been extended to take into account a probability distribution for the environmental variables. The underlying idea is to minimize a weighted average of the response over a discrete set of values for the environmental variables.

In the worst-case context, an algorithm has been proposed in [59] to deal with environmental variables for the robust optimization of black-box functions evaluated by costly computer experiments. It combines EGO with an iterative relaxation procedure, to provide an algorithm for the minimax optimization of black-box functions. Relaxation makes it possible to take into account continuous infinite spaces for both the control and the environmental variables, unlike the discrete probabilistic formulation mentioned before.

All of these algorithms have shown promising results and are especially well suited when a robust design is required for an expensive-to-simulate process, unlike most global-purpose optimization algorithms that require a thorough exploration of input spaces.

Sensitivity Analysis

Using black box or surrogate models, determining what the most influent inputs of a system on its output values are remains a key point. This sensitivity analysis deals with how the uncertainty in the output of a statistical model can be linked to different variations in the inputs of the model. Figure 7 illustrates an example of sensitivity analysis, applied to the study of space launcher stage fall-back positions. The color of the different samples varies with the input value. Inertial measurement appears to be the most influent factor on the fall-back position. Sensitivity analysis methods allow quantitatively similar results to be obtained in a rigorous manner. Two main approaches have been developed for that purpose.

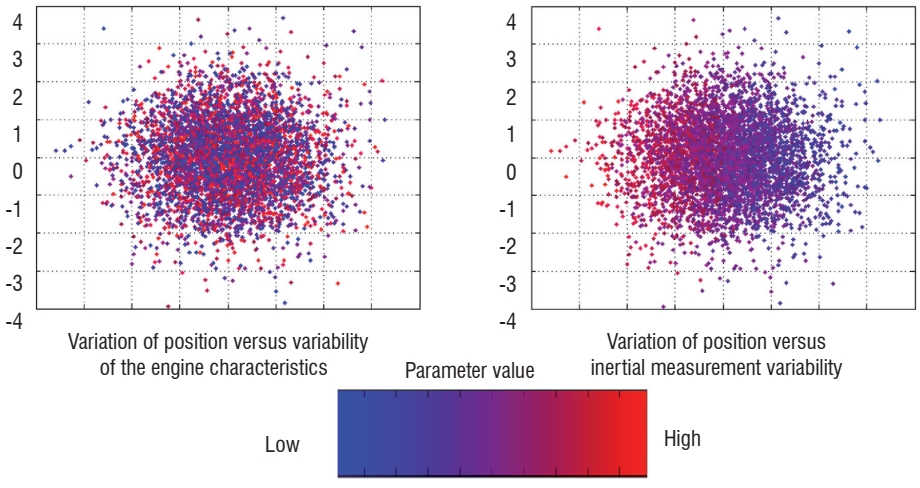


Figure 7 - Influence of two inputs on fall-back positions

Global sensitivity [40][96][87] analysis focuses on the variances of model outputs and more precisely on how the input variability influences the variance of a given output [38]. It makes it possible to determine which parts of the output variance are due to the different inputs, with the estimation of Sobol indices (see e.g. [95][86] for a detailed description). They are major tools in sensitivity analysis, since they give a quantitative and rigorous summary of how the different inputs influence the outputs.

Local sensitivity [67][1][79] analysis provides information on how small perturbations in a neighborhood of an input space value influence the model output value. Since local sensitivity analysis is not robust to non-linear effects and input interactions, it is often less used than global sensitivity analysis. Moreover, a global sensitivity method makes it possible to take into account the interaction between input values. Nevertheless, the local sensitivity analysis method has the advantage of requiring fewer simulations than the global method, when running the model is very time consuming.

Uncertainty propagation

Uncertainty representation under the form of a probability density function has been exploited in several methods for analyzing and propagating uncertainties in systems, such as interval analysis, Monte-Carlo and quasi Monte-Carlo techniques. However, none of these approaches is able to allow for the intrinsic nature (in terms of continuous random distributions) of the assumed stochastic uncertainties. To fill this gap in many methodologies, the Polynomial Chaos Theory (PCT) has been recently used to tackle this problem. This theory is based on the principles stated initially by Wiener in 1938 (see [106]) and then justified by Cameron and Martin in 1947 (see [7]). More recently, further research work has shown the effectiveness of the PCT in many engineering applications subject to stochastic uncertainties, such as mechanics, heat convection, fluid dynamics or automatics [24][30][55][103]. A recent wide survey of the theoretical background, including some practical results of the PCT, is available in [99]. The basic idea of the PCT consists in modeling any probabilistic uncertain system (static or dynamical) as an equivalent deterministic one, in a higher dimensional space, where the explanatory variables of the initial stochastic model are decomposed on a Hilbert basis of L^2, generated by the polynomial representation adopted to approximate any uncertainty of interest. This theory is a powerful alternative to Monte-Carlo and/or quasi Monte-Carlo methods in propagating probabilistic uncertainties and estimating more accurately the properties of any stochastic static or dynamical system, in terms of mean and variance.

An illustrative application of the PCT is provided in figure 8 , in which an AIRBUS A340-600 lateral flight dynamics linear model has been expanded over 3 uncertain aerodynamics derivatives. This figure compares the poles dispersion of the natural modes of the aircraft obtained by both Monte-Carlo and PCT methodologies and shows the relevance of the PCT for estimating the probabilistic properties of stochastic systems. In addition, the application of the PCT using Galerkin projections presents a significant computational advantage over Monte-Carlo and quasi Monte-Carlo methods.

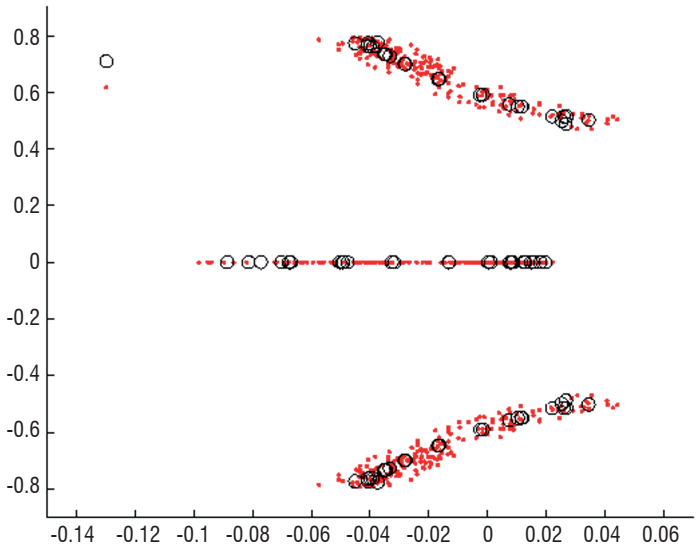


Figure 8 - Pole dispersions for Dutch Roll and Spiral stability modes Comparison between Monte-Carlo and PCT

Rare event estimation

Monte-Carlo or quasi Monte-Carlo representations are well suited for providing information on events whose associated probabilities are not too low. For very seldom observed events, such as the collision probability between two aircraft in airspace, these approaches do not lead to very accurate results. Indeed, the number of available samples is often insufficient to accurately estimate such low probabilities (at least 10^6 samples are needed to estimate a probability of order 10^{-4} with 10% relative error). It is therefore necessary to develop appropriate techniques to estimate these probabilities, requiring a fewer number of samples. They can be divided mainly into two categories: probability density function tail parameterization techniques based on extreme value theory and simulation techniques such as importance sampling or importance splitting.

Extreme value methods are very useful when it is not possible to obtain, or simulate, new samples. Finance [31] is the main domain of application of the extreme value theory, but some applications have also been proposed in the world of engineering [73]. There are two main kinds of model for extreme values. Block maxima models [25] [52][19] are notably used for the largest observations collected from a high number of identically distributed observations. It allows the law of the maximum of a sample collection to be determined. A more recent group of models is the peaks-over-threshold (POT) approach [18][60][72]. It allows the law of the samples to be determined conditionally to exceed a high threshold. POT models are generally considered to be the most useful for practical applications, due to their more efficient use of the (often limited) data on extreme values.

Simulation techniques require the ability to simulate new samples. Importance sampling is the most well-known rare event simulation technique. It is designed to reduce the variance of the Monte-Carlo

estimators for a given sample size. Importance sampling consists in generating random weighted samples from an auxiliary distribution rather than the distribution of interest. The crucial part of this algorithm is the choice of an efficient auxiliary distribution that must be able to simulate additional rare events. Various optimizations of auxiliary distributions have been described in [39][84][14][56][109][66][63]. Figure 9 shows different iterative importance sampling auxiliary densities for the $(1\text{-}10^{-7})$-quantile estimation of a centered reduced Gaussian density. The principle of importance splitting [47][9][8][64] is quite different. Instead of estimating one probability through a very costly simulation, one considers the estimation of several conditional probabilities that are easier to evaluate by simulation. The sought probability is then obtained with the use of the Bayes theorem. Importance splitting is notably very adapted when the simulation budget is important and the probability to be estimated is very low ($<10^{-6}$).

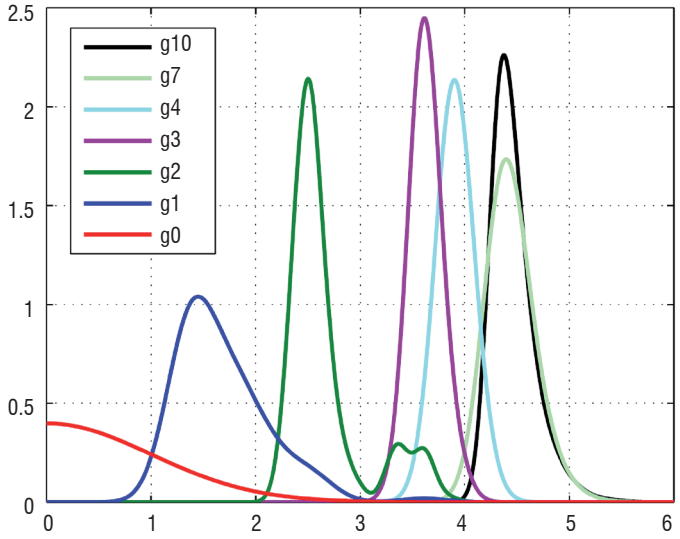


Figure 9 - $(1\text{-}10^{-7})$-Quantile estimation of centered reduced Gaussian variable with non-parametric adaptive importance sampling

Application areas of rare event estimation methods in reliability and safety engineering are very broad and are not restricted to aerospace. Among the applications currently dealt with at Onera, figure 10 illustrates the fall-back zone estimation of a space launch vehicle [65] and figure 11 presents the collision probability map estimation between aircraft in uncontrolled airspace with rare event methods.

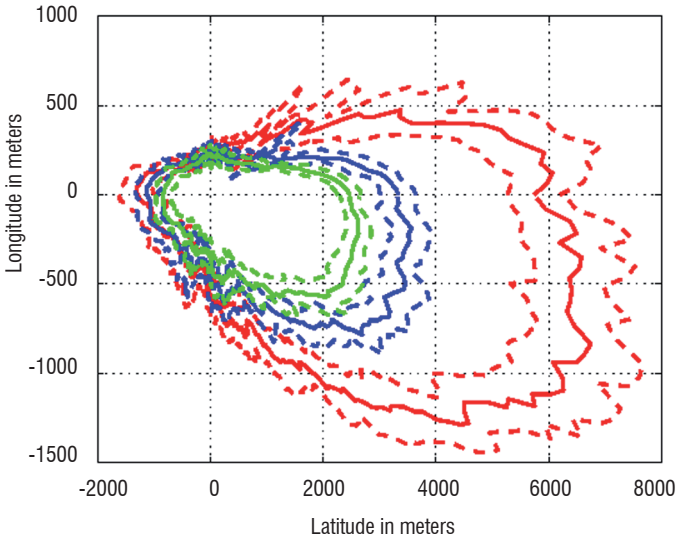


Figure 10 - Mean (solid line) and standard deviation (dashed line) of a launcher stage fall-back zone with importance sampling simulations (95 % =green, 99% =blue and 99.99% =red)

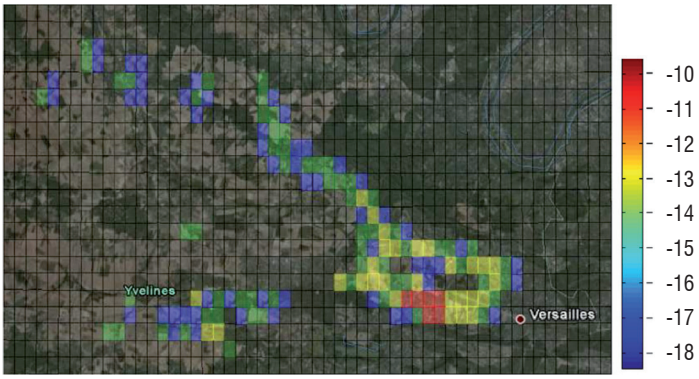


Figure 11 - Map of collision probability estimates with importance splitting. The color scale depends on the logarithm of the probability

Box 2 - Estimating satellite versus debris collision probabilities via the adaptive splitting technique

Spacecraft collision still does happen seldom, but the loss of a satellite cannot be afforded. This high risk therefore must be addressed carefully. To support the decision to start a collision avoidance maneuver, a dedicated tool is the probability of collision between the debris and the satellite [44]. Crude Monte-Carlo could be a way, if it could cope with very small probabilities, say 10^{-6}, within the available simulation budget and time. The methodology in use nowadays is a numerical integration made tractable by physical hypothesis and numerical approximation [10]. An efficient alternative is to consider an importance splitting technique, since this avoids the different hypotheses needed for the numerical integration and clearly outperforms CMC with respect to rare events, as is illustrated in table B2-01. However, importance splitting requires tuning. Some experience based empirical tuning rules are notably proposed in [71].

	Collision probabiity estimate (relative error)	Mean sample number
Monte-Carlo	$1.1\ 10^{-6}$ (172%)	300000
Importance Splitting	$1.9\ 10^{-6}$ (32%)	309000

Table B2-01- Importance splitting-Monte-Carlo comparison

Time-dependent system analysis

Performance evaluation can also be sought for a time-dependent system. Examples of such systems include the weather evolution, stock market, plane trajectories. In the following, different methods to deal with time-dependent stochastic process of complex systems are presented.

Multi-model representation for performance analysis

The first approaches for characterization and analysis of dynamical system performances have been derived from the safety literature. Most of them are based on combinatorial approach such as fault or event trees or reliability diagram (see e.g. [42][82]). Using these methods allows the combination of events that lead to a specific sys-

tem behavior to be identified. However, basic fault trees are not well suited for modeling complex systems with strong dependencies between the system components. The assumption of component independence is a powerful feature of fault trees, but is extremely restrictive and may lead to erroneous results.

In order to provide a realistic evaluation of the system performances, the model should include a representation of the dynamical interaction between its physical parameters and the functional behavior of its components. In the 80's, Davis presented the Piecewise Deterministic Markov Processes (PDMP) [15][17][16]. This class of models constitutes a well-suited framework to represent dynamical systems subject to (expected or unexpected) changes in their behaviors. The description of the system is provided in the form of a set of small models, each related to a different operating behavior mode [108]. The representation is based on two types of variables, one representing the dynamical states of the system and the other representing the mode, usually with discrete values. The states are used for the deterministic representation of the system evolution, whereas the mode indicates the suitable characterization of its behavior. The change in the functioning is reflected in the modification of the mode value.

Representing a complex system under this form requires, firstly, to split it into subsystems or components. It is then possible to identify all of the possible regimes and dynamical models for the subcomponents and to define whether the change of mode from one behavior to the next is deterministic or probabilistic. The probabilistic jumps can be modeled using a Poisson law and the deterministic jumps are obtained for a given time, or when the state components satisfy a predefined criterion. Given a time period, it becomes possible to simulate the whole trajectory of the entire system. Evaluating a performance criterion is then performed by simulations, based on indication on the characteristics of the Poisson law and the modeling of the piecewise deterministic representation. This modeling can also be used to determine suitable control, for example, for reconfiguration purposes [13].

Markov Chain

Another representation of the dynamic evolution of a random process is a Markov chain, which consists in a collection of random variables (X_n where n is a positive integer)[17][61]. Namely, if X_o is the initial state of the process, X_n represents its state at the n^{th} step, the result of n random moves (transitions) in the state-space. In engineering applications, steps are usually taken as a measure of time.

A Markov Chain usually refers to a discrete random process, with the Markov property stating that the conditional probability distribution for the system at the next state, given its current state, depends only upon the current state (i.e. the process has no memory). If P is the probability distribution for the system at a given state, to move to the next one (the transition probability), the following relation holds (according to the Markov property):

$$P\left(X_{n+1}=x_{n+1}\middle|X_1=x_1,\ldots,X_n=x_n\right)=P\left(X_{n+1}=x_{n+1}\middle|X_n=x_n\right)$$

A Markov chain is fully characterized by its state-space and transition probabilities.

Among specific properties, usually highlighted, a Markov chain is said to be irreducible if it is possible to get to any state from any state; it is said to be k -*periodic* if any return to a given state occurs in multiples of k steps; a state is said to be *transient* if the probability that the trajectory of the random process starting from this state has a non-zero probability of never going back to this state (otherwise, the state is *recurrent*); a state is said to be ergodic if it is aperiodic and recurrent (with a finite mean recurrent number of steps).

Finally, it is usually impossible to predict the exact trajectory of a Markov chain, since it is a random process, nevertheless in specific case it is possible to predict the statistical properties of the Markov sequence. For example, an irreducible and aperiodic Markov chain will reach a stationary probability distribution $\pi\left(i.e.\ X_n \sim \pi\right)$. Many algorithms allow Markov chains with the desired statistical properties to be constructed, thus offering the ability to generate sequences of random numbers that reflect targeted distributions. These processes (namely Monte-Carlo Markov Chains) have been very useful in Bayesian inference and many other fields. An example of a Markov chain is given in figure 12 to analyze the random trajectory of a commercial flight [41][78].

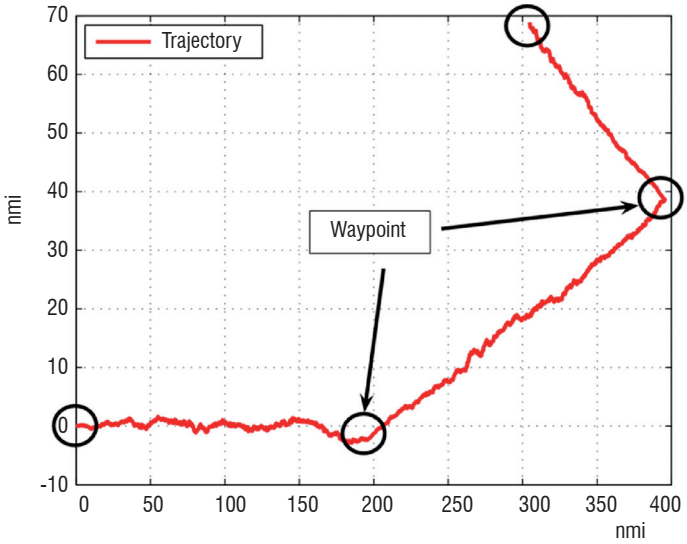


Figure 12 - Commercial flight trajectory modeled with a Markov chain

Bayesian Networks

The Bayesian Network (BN) theory is a formalism based on the Graph Theory and the Bayes probabilistic Theorem. It is used for representing systems or entities in a common framework. More specifically, using a Bayesian Network is a way of describing any system or entity through a structured probabilistic approach based on both qualitative and quantitative information. It brings the ability to unify various kinds of knowledge into a single representation. On the one hand, qualitative considerations are translated into a specific Directed Acyclic Graph, while, on the other hand, quantitative data mining makes it possible to parameterize and quantify the correlation links making up the network, through a Joint Probability Distribution.

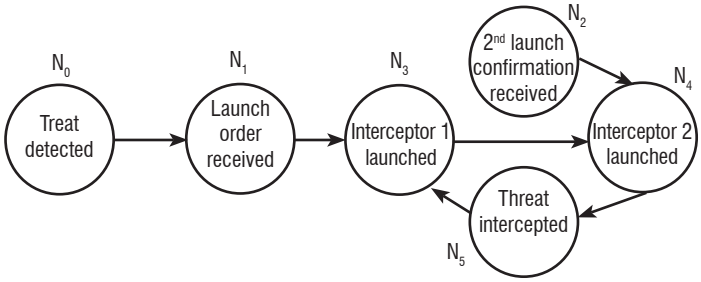


Figure 13 - Example of Bayesian Network used to model a threat interception by two interceptors

Directed Acyclic Graphs (as illustrated in figure 13) consist of nodes and directed arcs. Nodes represent discrete random variables, while the arcs show the causal or statistical interaction between the variables. These diagrams describe a set of conditional independence assumptions corresponding to the graph theoretic notion of d-separation. If two nodes X and Y are d-separated, (i.e., if every path between X and Y is "blocked" by a 3rd variable Z), then X and Y are totally independent, given Z [36] (for example, in figure 13, N_1 and N_5 are independent, given N_3). Whereas two connected nodes have direct causal relationship and one can specify the local conditional probabilities in a Conditional Probability Table (CPT) for each node. For a given node, the CPT aggregates, for each possible state of the variable associated to this node, all of the conditional probabilities with respect to all of the combinations of values of the variables associated to each parent node. Nodes with no parents (N_0, N_2 in figure 13) are called root variables and their CPTs are filled up with marginal "a priori" probabilities.

Under these assumptions, the joint probability distribution P of random variables (X_i) can be factorized as follows:

$$P\left[X_1, X_2, ..., X_n\right] = \prod_{i=1}^{n} P\left[X_i \mid Parent\left(X_i\right)\right]$$

In the case of the BN in figure 13, the joint probability distribution of the random variables X_0, X_1, X_2, X_3, X_4, X_5 is hence given by:

$$\begin{aligned} P\left[X_0, X_1, X_2, X_3, X_4, X_5\right] = {} & P\left[X_0\right].P\left[X_2\right].P\left[X_1 \mid X_0\right] \\ & .P\left[X_3 \mid X_1\right].P\left[X_4 \mid X_2, X_3\right] \\ & .P\left[X_5 \mid X_3, X_4\right] \end{aligned}$$

The information required to build the network structure and to provide prior probabilities and Conditional Probability Tables can be supplied by expert knowledge or by feedback on the use of the physical system itself. With no observation, the computation is based only on a priori probabilities. Any new observation, called evidence, enhances the accuracy of the system BN description.

Inference algorithms designate the mechanisms used to propagate the probabilistic information throughout the entire network. These consist in computing probabilities for all of the unmeasured nodes, given the information available about the states of the observed nodes (evidence). Algorithms may be sorted into two main families. The first one encompasses the so-called exact inference methods. Ref. [46] and [90] address the Clustering algorithm, which uses the conditional independence properties contained within the network to compute a posteriori exact probabilities. The second one relates to approximate inference algorithms, such as Logic sampling [37], Likelihood weighting [27], Backward sampling [28], Self-importance [91] and Heuristic importance [91]. These algorithms estimate probabilities through multiple draws among the set of possible states for each variable of the network.

Different extensions to BN have been proposed to handle, for example, temporal aspects, logical relations, or the modeling of complex systems.

For a complex system composed of a high number of components or subsystems, the structure of the corresponding BN can be hard to deal with. Object Oriented Bayesian Networks (OOBN) [105] provide a methodological framework to decompose a complex BN into several hierarchical layers. Therefore, a complex system can be more easily modeled by an OOBN from functional analysis and decomposition into physical components.

To model dynamical systems, a temporal dimension can be added to BN, leading to Dynamical Bayesian Networks (DBN) [68] where random variables are indexed by time. Information about the initial state $\left(X_1(0), ..., X_n(0)\right)$ is given through a prior probability $P\left[X_1(0), ..., X_n(0)\right]$. A BN is then used to represent the transition probability by $P\left[X_1(t), ..., X_n(t) \mid X_1(t-1), ..., X_n(t-1)\right]$ defining the temporal probabilistic dependencies between nodes of two time slices. In the particular case of dynamical systems, for which the state evolution can be represented by a discrete time equation, the physical knowledge modeled by this state equation is directly taken into account in the formulation of the conditional probabilities associated to each random variable.

ATLAS method

ATLAS (Analysis by Temporal Logic of Architectures of Systems) is a method developed at Onera aimed at providing a quick macroscopic tool for the probabilistic performance assessment of time-dependent systems.

The most notable frameworks dealing with stochastic approaches for time-dependent systems are generally based on one of the three following approaches [5]: Bayesian networks [48], described in the previous section, stochastic Petri nets [23][70] and fault trees [21] [101] or related formalisms [10][98]. Among these approaches, those based on stochastic Petri nets, though interesting, require heavy simulation (combined for example with Monte-Carlo methods). Although the Bayes network approach is interesting, large Bayesian networks reflecting complex systems are difficult to design and maintain. Finally, fault trees consist in a method in which the potential causes of a system hazard are recursively organized into a tree structure reflecting causality - which is a crucial notion in the framework of safety analysis - trying to figure out all of the credible ways in which the hazard may occur.

The ATLAS approach is dual here and does not focus on the reliability of the system, but rather on its performance - although both are of course related. Therefore, one does not consider the sets of causes that lead to a failure, but rather those that lead to a success. This is determined from the functional analysis of the system. Temporal consistency is represented by a modal logic, allowing the expression of time. This can be linked to the fault tree approaches using time propagation such as [35][89] based on the Interval Temporal Logic and the Duration Calculus Interval ([2][110][33][34]).

Contrary to these approaches, the ATLAS method simultaneously combines the time and probability aspects. It was first introduced to evaluate the performance of a defense system [96] and extended to other fields of applications. The detailed method description can be found in [6], [50] and [49]. It was also successfully applied to system analysis in various contexts, such as ballistic missile defense performance assessment [3] and the assessment of space system vulnerability to debris [51].

The determination of the probability of success and necessary delay of a given dynamic system using the ATLAS method is analyzed using the following steps. The system is described as a tree-shaped

temporal combination of elementary functions (see figure 14) with a dynamic component.

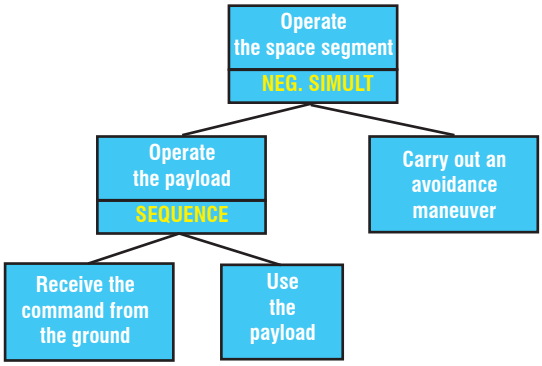


Figure 14 - A tree reflecting the (partial) behavior of a space system

In this tree, the leaves, which are said to be “elementary functions”, are distinguished from other functions. They are assumed to be pairwise independent and they represent the input point of ATLAS, in the form of the availability performance defined in the next subsection.
The definition of this tree is a key point of ATLAS and must include the two following features: first, the identification of independent elementary functions and, second, temporal links between these functions. The first feature is generally done by functional analysis, the depth of the decomposition depending on the purposes of the study. The second one is generally done using temporal flow diagrams, using a notion of triggering and concluding events, where patterns corresponding to operators are identified (see [49] for further details). How such a tree may be obtained is illustrated in box 3.

In order to take into account the temporal aspect of the performance, the success probability associated with each function is represented as a function of its starting times $(s_i)_{1\le i\le m}$ and final times $(t_j)_{1\le i\le n}$. This representation, called the function availability performance, is illustrated in figure 15 , where the abscissa represents the initial time; the ordinate represents the final time and the applicate represents the probability.

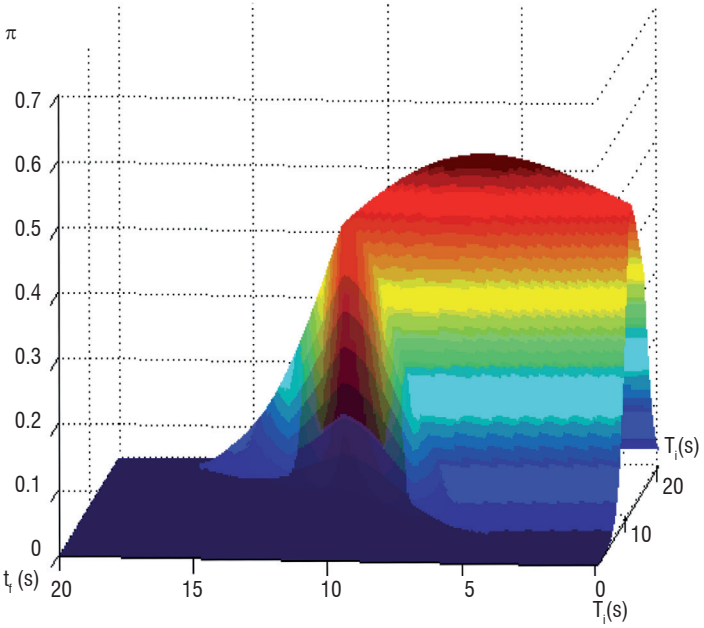


Figure 15 - A function availability performance

Box 3 - Using ATLAS for the assessment of a Ballistic Missile Defense system

Ballistic missile defense (BMD) architecture performance (threat neutralization probability) evaluation is a complex problem. It is therefore of interest to evaluate it without using Monte-Carlo simulations that may prove very costly. The system is described in figure B3-1.

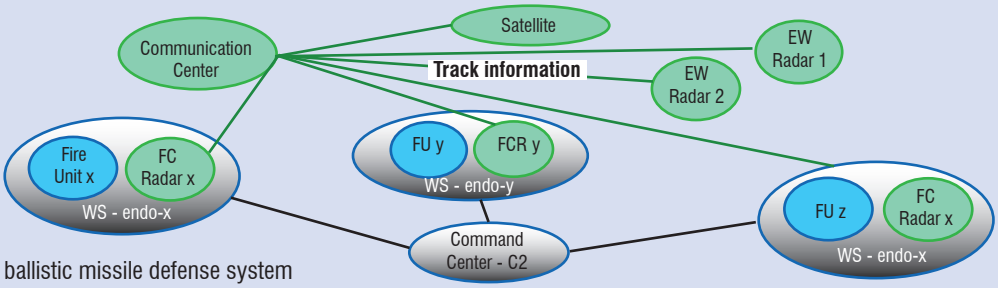
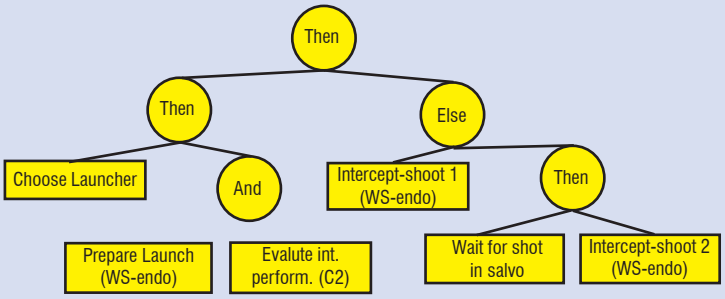
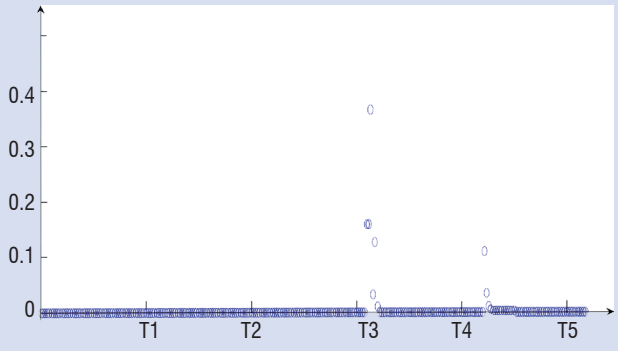


Figure B3 - 1- A ballistic missile defense system

Using the ATLAS methodology, the elementary functions consist, for example, of “Choose launcher” or “Wait for shot in salvo”, as presented in figure B3-2 . Elementary temporal probabilities have been provided by experts from the relevant domain.





Since it is a discrete probability of success of the elementary function, this performance notion satisfies: for any s_i, $\Sigma_{1\leq j\leq n}\pi^{F}\left(s_i,t_j\right) \leq 1$ (as one considers potential failure, this sum may be strictly less than 1). This accumulation represents the overall probability of success of a function F, which could be expressed for all combination of m starting time and n end time by $\Sigma_{1\leq i\leq m}\Sigma_{1\leq j\leq n}\pi^{F}\left(s_i,t_j\right) / m$.

Another way to view this performance is to consider that a service consists in answering an order given at some instant, within a specific answer delay, which leads to the following performance notion: $\pi^{F}(s,t)$, which represents the probability that function F will provide, at time t, an answer to an order received at time s. In this sense, the performance may be seen as a probability of t being a final time value for each given initial time.

Once the user has defined these characteristics for elementary functions, the purpose of ATLAS is to combine them into a temporal logical way, satisfying the linking constraints expressed by the tree, in order to obtain the availability performance for the full system and the intermediate nodes (see [49] for further details). These performances may be calculated quickly over a large family of scenarios, allowing post-treatments suited to the purposes of the study.

ATLAS is an innovative approach for system assessment, interestingly combining temporal and probabilistic aspects. It has been compared to Monte-Carlo simulations and to dynamic Bayesian networks, and has proved to provide the same results but within a shorter time, since the probability for each pair of starting and finishing times is already set. Its system-engineering-oriented point of view facilitates its use by system specialists and its output for the expected performance at each node of a functional decomposition is also a potentially valuable tool for system design.

New notions are currently considered for use in the ATLAS framework, such as resource consumption evaluation.

Conclusion

In this article, we have presented a large spectrum of methods for evaluating performances of complex systems that could significantly improve the results obtained with classical algorithms. It is nevertheless sometimes difficult to determine a priori which techniques would be the most efficient for a particular case. A major feature is the type of knowledge available on the system behavior. Otherwise, simulation budget, dimension of the problem or density models also play an important role in the selection of the most efficient and suitable methods. Previous analysis of the complex system characteristics must thus be performed before choosing an appropriate approach ■

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Acronyms

CEP (Circles of Equal Probability)
EGO (Efficient Global Optimization)
PCT (Polynomial Chaos Theory)
POT (Peaks-Over-Threshold)
PDMP (Piecewise Deterministic Markov Processes)
BN (Bayesian Network)
ATLAS (Analysis by Temporal Logic of Architectures of Systems)
CMC (Crude Monte-Carlo)
BMD (Ballistic Missile Defense)

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