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Transition and Turbulence Modeling

Although the basic equations of fluid motions have been well known for long, their complete solution in practical applications is beyond the scope of present and foreseeable computers. Models are thus required to account for the transition to turbulence mechanisms as well as to represent at least a part of the turbulent motion. This paper aims at giving an overview of the present modeling practices and the models developed or imported by Onera and implemented in the CEDRE and/or *elsA* solvers.

Introduction

Fluid motions are fully described by the Navier-Stokes equations which express the conservation of mass, momentum and, if needed, energy and/or chemical species. From a theoretical point of view, solving these equations remains a challenge beyond the present capabilities of mathematicians and is one of the Millennium Problems proposed by the Clay Mathematics Institute. From an engineering point of view, practical flows in aerospace applications are mostly turbulent. As turbulence is characterized by a large variety of scales (see e.g. [40]), all these scales must be captured in the flow computation. This is the Direct Numerical Simulation approach (DNS) which is far beyond the capabilities of present and foreseeable computers. Extrapolating the progress in computer power and computer sciences, it is expected that the DNS computation of the flow around an airliner in cruise conditions or around a car on a (US) highway could be feasible circa 2080 [77]. Therefore, only a simplified vision of the flow can be computed and models are required to represent the part of the physics which cannot be resolved. This holds both for the prediction of the flow instabilities which lead to the transition from the laminar to the turbulent regime and for the fully turbulent regime. Transition and turbulence modeling aspects are detailed below.

Transition modeling

Since the classical experiments performed by Reynolds [65], constant interest has been shown in the instability of laminar flows and the transition to turbulence for solving fluid mechanics problems. This interest results from the fact that transition controls important hydrodynamic quantities such as drag or heat transfer. For instance, the heating rates generated by a turbulent boundary layer may be several times higher than those for a laminar boundary layer; therefore transition prediction is of great importance for hypersonic re-entry vehicles. In the case of commercial transport aircraft at high subsonic speed, the achievement of laminar flow can significantly reduce the drag on the wings and hence the fuel consumption of the aircraft.

When a laminar flow develops along a given body, it is strongly affected by various types of disturbances generated by the model itself (roughness, vibrations…) or existing in the free-stream (turbulence, noise…). These disturbances are the sources of complex mechanisms which ultimately lead to turbulence. There are in fact two main paths to turbulence:

- If the laminar boundary layer develops on a "perfectly smooth" wall, in a low free-stream disturbance environment (for instance in flight conditions), transition results from the amplification of unstable waves: this process is called "natural transition".
- In the presence of strong disturbances (high free-stream turbulence, large roughness elements), these waves are no longer observed. In this case, streamwise streaks appear and play a major role in the transition process; this mechanism has been named "bypass" by Morkovin [54].

Both aspects will be analyzed successively in the following paragraphs.

Natural transition

General description

To describe the laminar-turbulent transition process in two-dimensional (2D) or three-dimensional (3D) boundary layers, it is helpful to distinguish three successive steps, as illustrated in Figure 1. The first step, which takes place close to the leading edge, is the *receptivity*. Receptivity describes the means by which forced disturbances such as free-stream noise or free-stream turbulence enter the laminar boundary layer and excite its eigenmodes. In the second phase, these eigenmodes take the form of periodic waves, the energy of which is convected in the streamwise direction. Some of them are amplified and will be responsible for transition. Their evolution is well described by the linear *stability* theory. When the wave amplitude becomes finite, nonlinear interactions occur and lead rapidly to turbulence.

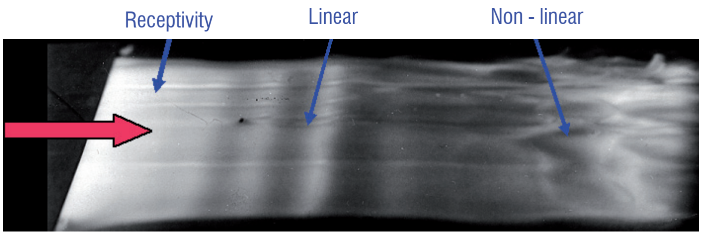

Figure 1 - "Natural" transition on a 2D flat plate, visualization in water channel (Werlé, Onera).

In 2D flows, the linearly growing waves are referred to as Tollmien-Schlichting (TS) waves. In 3D boundary layer flows, for instance on a swept wing, the mean velocity profile has two components: a streamwise component u in the external streamline direction, and a cross-flow component w in the direction normal to the previous one. The streamwise velocity profile is unstable in regions of zero or positive pressure gradient (decelerated flows). It generates waves similar to the 2D TS waves, with a wave number direction close to the free-stream direction. The cross-flow velocity profile is highly unstable in negative pressure gradients (accelerated flows). It generates cross-flow (CF) waves with a wave number vector making an angle of 85 to 89º with respect to the free-stream direction.

As it will be explained later, the receptivity process and the nonlinear interactions are different for TS and CF disturbances. In the linear phase, however, the same stability theories are applicable for both types of waves.

Natural transition modeling

In the framework of the classical linear stability theory, the disturbances are written as:

$$r' = \hat{r}(y)\exp\left[i\left(\alpha x + \beta z - \omega t\right)\right] \quad (1)$$

r' is a velocity, pressure or density fluctuation; $\hat{r}$ is an amplitude function; x and z are the directions normal and parallel to the leading edge, y is the direction normal to the wall. When considering the spatial theory (which is the most relevant for a wide range of boundary layer problems), $\alpha = \alpha_r + i\alpha_i$ is the (complex) wave number in the x direction. β and ω are real and represent the wave number component in the *z* direction and the frequency. The angle ψ defined as:

$$\tan\psi = \beta / \alpha_r \quad (2)$$

represents the wave number direction, i.e. the direction normal to the wave crests.

Introducing expression (1) into the linearized Navier-Stokes equations and assuming that the mean flow is parallel, leads to a system of ordinary differential equations for the amplitude functions (eigenvalue problem). For the simplest case of a 2D, low speed flow with $\beta = 0$, the stability equations can be combined to obtain the well-known Orr-Sommerfeld equation. Depending on the value of ψ, the solutions of the linear stability equations represent either TS or CF waves.

Non linear PSE (Parabolized Stability Equations) can be used in order to model the *non linear interactions* between waves just before the breakdown to turbulence, see [37]. The disturbances are now expressed as a double series of (n,m) modes of the form:

$$r' = \sum_{n=-\infty}^{n=+\infty}\sum_{m=-\infty}^{m=+\infty} \hat{r}_{nm}(x,y)\exp\left[i\left(\int \alpha_{nm}(\xi)d\xi + m\beta z - n\omega t\right)\right]. \quad (3)$$

As for the linear theory, α_{nm} is complex; β and ω are real numbers. Each mode is denoted as (n,m); the integers n and m characterize the frequency and the spanwise wave number, respectively. When these disturbances are introduced into the Navier-Stokes equations, a system of coupled partial differential equations is obtained; this (nearly) parabolic system is solved by a marching procedure in the x-direction. Any non-linear PSE computation requires a choice of the "most interesting" interaction scenario between particular modes ("major modes") and imposition of initial amplitudes A_{in} for these modes. The numerical results show that the non linear behaviors are different depending on the nature of the dominant instability at transition: in the case of TS waves, resonances between 2D $(\beta = 0)$ and oblique modes lead to an sudden increase of the mode amplitudes; in the case of CF waves, a saturation is observed before the breakdown to turbulence.

As far as the receptivity process is concerned, a distinction must be made between TS and CF instabilities:

- TS instability is very sensitive to the free-stream disturbances (noise or the free-stream turbulence), which are usually quantified by the non-dimensional parameter Tu.
- The CF waves cover a wide frequency range. In particular, zero frequency waves are highly amplified by the cross-flow mean velocity component w. They take the form of stationary vortices nearly aligned with the external streamlines. At low Tu, these vortices play the major role in the transition process by creating a steady inflection point on the streamwise velocity profile. It is now recognized that the source of the CF vortices lies in the micron-sized roughness elements (i.e. the surface polishing) at the location where the vortices start to be amplified [61], typically between 1 and 5% chord on a swept wing. It follows that improving the surface polishing of the leading edge decreases the initial amplitude of the vortices and delays transition.

Natural transition prediction

The most popular method for predicting transition is the e^N criterion, developed more than 50 years ago by Smith and Gamberoni [74] and by van Ingen [84], see review in [4]. The so-called N factor is the total growth rate of the most unstable disturbances. For the simplest case of 2D, incompressible flows, it is computed by integrating $-\alpha_i$ in the x direction. The procedure becomes more complicated for compressible and/or 3D flows, but the principle remains the same. Transition is assumed to occur for some specified value N_T of N; for instance, N_T lies in the range 8-10 on 2D airfoils in low turbulence wind tunnels. The e^N method is based on the linear theory only and does not take the receptivity and the non linear mechanisms into account explicitly.

As the use of the e^N method is often time consuming, the development of simplified methods is of unquestionable practical interest. The simplest solution is to apply analytical criteria expressing relationships between boundary layer integral parameters at the transition point, see for instance [53], [35], [3]. The latter criteria have been implemented in the *elsA* code. Another possibility is to use simplified stability methods (the so-called "database methods"), the complexity of which is intermediate between analytical criteria and exact stability computations [59].

The above methods, however, are not well adapted to massively parallel RANS computations because they use non-local quantities such as momentum thickness or shape factor. To avoid these difficulties, Menter [52] proposed a purely local transition model which consists of two transport equations for the intermittency function and for a pseudo-momentum thickness Reynolds number, coupled with a SST $-k\omega$ turbulence model. At Onera, this model has been implemented in the *elsA* code. Once calibrated by comparison with experimental data or stability results, it gives satisfactory results for 2D flows, see [23]. It does not include the cross-flow instability.

At first sight, the non linear PSE could be considered as the most rigorous tool for transition prediction, because they describe both the linear and the non linear developments of the unstable waves. However, it is important to keep in mind that the abscissa corresponding to the resonance (for TS waves) or to the mode saturation (for CF waves) depends on the initial amplitude A_{in} imposed on the major modes. Increasing A_{in} leads to an upstream movement of this abscissa and of the numerical breakdown location. In other words, the choice of the N factor, which constitutes the major difficulty of the linear e^N method, is now replaced by the choice of A_{in}. Therefore the non linear PSEs cannot yet be considered as mature enough for practical transition prediction. A measure of the receptivity is needed.

Bypass Transition

General description

In many practical situations, laminar-turbulent transition occurs at lower Reynolds numbers than those predicted by the classical linear stability theory. This suggests that another transition mechanism may exist. This process, called "transient growth", results from the non-normality of the eigenfunctions (solutions of the linear stability equations): if two eigenfunctions are not orthogonal, the perturbation energy of their sum can increase even if both of them are damped. The physics of the transient growth is the following. A longitudinal vortex superimposed to the boundary layer shear stress pushes up low speed particles from the wall to the top of the shear layer, and pulls down high speed particles toward the wall, leading to a spanwise alternation of low and high speed streamwise structures called streaks. This phenomenon was called "lift-up" by Landahl [43]. In other words, as soon as longitudinal vortices are present in a laminar boundary layer, streaks are likely to appear rapidly downstream. An early laminar-turbulent transition can be triggered if the energy of the streaks grows significantly; this is the "bypass" transition process, meaning that the classical process driven by the TS or CF waves has been short-circuited.

Bypass transition can be observed when the laminar boundary layer is subjected to a large free-stream turbulence level Tu (typically larger than 1%). The longitudinal vortices which initiate the transient growth process are generated by the large structures of the free-stream turbulence. In this case, the streaks are called "Klebanoff modes". An example of smoke visualization for a flat plate boundary layer with $Tu = 6\%$ is shown in the left hand part of Figure 2 [49]. The streamwise streaks are clearly visible. Note that transition is out of the figure.

Recent studies have demonstrated that streamwise streaks also play a significant role in the transition process downstream of an isolated roughness element. An excellent review of the recent experimental investigations on this subject has been given by Ergin and White [32]. The flow about an isolated 3D element consists of a steady horseshoe vortex wrapped around the upstream side of the obstacle, with two steady counter-rotating legs trailing downstream. These steady disturbances evolve rapidly downstream into low- and high-speed streaks aligned with the flow direction. Transition is defined as the location where a turbulent wedge starts to develop with a half-angle around 10°, see right hand part of Figure 2. At sufficiently high Reynolds numbers, *unsteady disturbances* (often associated with hairpin vortices) originate from the separated region just aft of the roughness element and can contribute to the transition process.

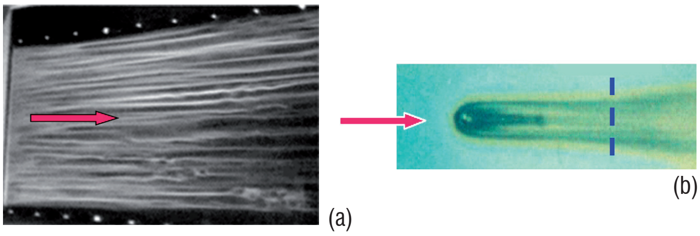

Figure 2 - Examples of bypass transitions.
(a) Transition induced by a high free-stream turbulence level.
(b) Transition induced by an isolated roughness element.

Bypass transition modeling

The linear development of the streamwise structures responsible for bypass transition can be described by an optimal growth theory, in which the disturbances are written as:

$$r' = \hat{r}(x,y)\exp\left[i\left(\beta z - \omega t\right)\right]. \tag{4}$$

Again r' is a velocity, pressure or density fluctuation; $\hat{r}$ is an amplitude function; ω is the frequency and $\beta = 2\pi / \lambda_Z$ the spanwise wave number (λ_Z is the spanwise wavelength).

By contrast with the classical linear theory, the length scales are different in the x, y and z directions. The streamwise coordinate x is made dimensionless with a characteristic length L (for instance the length of a flat plate or the chord of an airfoil) and the corresponding velocity u' is scaled with the free-stream velocity U_e. The y and z coordinates, on the other hand, are made dimensionless with the usual boundary layer scale $l = \sqrt{\nu L / U_e}$ and the corresponding velocities v' and ω' are scaled by U_e / Re_l, with $Re_l = U_e l / \nu$.

These boundary layer-type approximations lead to a parabolic system which can be solved by a marching procedure in x with initial conditions imposed at the starting location x_0. The objective is to maximize the growth G of the disturbance energy between x_0 and some downstream position x_1. This can be done by solving the direct system (from x_0 to x_1) and the adjoint system (from x_1 to x_0) iteratively, see details in [45][14][85] for instance. The numerical results show that the maximum growth is obtained with streamwise vortices as initial disturbances ($u' << v'$ and ω') and streamwise streaks at the final station ($u' >> v'$ and ω'). In addition the frequency with the highest amplitude is $\omega = 0$, i.e. the disturbances are steady. These results are in qualitative agreement with the experimental observations.

As stated before, the optimal growth (or transient growth) theory is linear. A complete picture of the bypass transition phenomena also includes a modeling of the receptivity process and of the non linear phenomena leading to the breakdown to turbulence. It is not yet clear today if the receptivity mechanisms generating the streaks are linear or not. Concerning the non linear final stage, it seems that streaks of sufficiently large amplitude become unsteady and that the breakdown to turbulence results from a sudden Kelvin-Helmholtz instability.

Bypass transition prediction

Many empirical criteria have been developed for many years in order to predict the occurrence of bypass transitions. For instance, the effect of high free-stream turbulence levels is taken into account by the well-known correlation proposed by Abu Ghannam and Shaw [1]. Concerning the problem of boundary layer tripping by large, isolated 3D roughness elements of height k, a relevant parameter is a characteristic Reynolds number Rk defined as:

$$Rk = \frac{U_k k}{\nu_k} \tag{5}$$

U_k and ν_k denote the mean velocity and the kinematic viscosity at the altitude $y = k$. These values are computed in the undisturbed flow. Von Doenhoff and Braslow [87] developed an empirical correlation between the critical value of Rk (denoted as Rk_{crit}) which triggers transition and the ratio d/k, where d is a measure of the spanwise or chordwise extent of the protuberance (for circular cylinders normal to the wall, d is the diameter). Rk_{crit} is of the order of 500-600 for $d/k = 1$ and 200-250 for $d/k = 10$. Systematic applications of this criterion showed that it remains valid for a wide range of applications, in 2D and 3D flows, from subsonic to supersonic flows.

The transition model proposed by Menter et al. [52] can also predict transition in the presence of large values of Tu. Examples of applications for turbo-machinery problems using the RANS code els*A* can be found in [11].

Quite recently, attempts have been made to use the transient growth theory in order to quantify or to predict the effects of roughness on transition. Following the work of Luchini [45], Vermeersch [85][86] developed a system of parabolic, linear transport equations for the streamwise velocity and temperature fluctuations of the streamwise streaks. To close the system, the vertical velocity fluctuation v' is modeled by an analytical relationship. Transition is assumed to occur when the ratio between the shear stress generated by the streaks to the viscous stress reaches some predefined critical value. This model was successfully applied to bypass transition problems, both in the case of large free-stream turbulence levels and in the case of boundary layer tripping by large roughness elements. In the latter case, it was possible to determine a theoretical curve for Rk_{crit} as a function of d/k. This curve was in good agreement with the von Doenhoff and Braslow criterion mentioned above. This confirms that the transient growth theory contains (at least a part of) the physics of the boundary layer tripping mechanisms.

Outlooks

After more than fifty years, the e^N method remains the most widely used method to estimate the "natural" transition location, although its deficiencies are well identified: the receptivity mechanisms are not accounted for explicitly and the nonlinear phase is replaced by a continuous linear amplification up to the onset of transition. The nonlinear PSE equations, on the other hand, are now a classical research tool. Although they cannot be used for systematic practical applications, they are a help in understanding the basic phenomena leading to transition. As much information has been collected during the last ten or fifteen years on receptivity, a rather complete but partly empirical modeling of "natural" transition is now available.

The state-of-the-art for the modeling of bypass transition is not so advanced. Most of the practical prediction methods used today are based on simple criteria, which ignore the complicated physics of the phenomena. Recent investigations have shown that the linear transient growth theory appears to be an efficient tool for the understanding and the modeling of these phenomena. The validity of this approach needs to be validated in complex situations, in particular for 3D and/or compressible flows. In addition, the picture of the receptivity and non linear mechanisms has to be completed.

Turbulence modeling

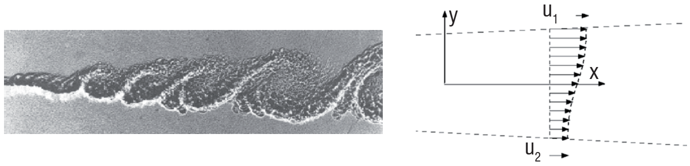

Figure 3 - Instantaneous (left, from [16]) and averaged (right) visions of the mixing layer between two parallel flows of different velocities.

Reynolds [66] proposed decomposing the flow into a mean motion, defined as an ensemble-average (and in most cases, a time average), and turbulent fluctuations. This leads to the Reynolds Averaged Navier-Stokes (RANS) equations. This mean motion was at that time what was measured by most sensors such as pressure probes which naturally time average. However, this average motion has no real existence and cannot really be seen, except using averaging sensors. Figure 3 points out the difference between the real flow and the time-averaged flow in the simple case of a mixing layer. Big, highly corrugated, rollers are visible in one case, completely smeared out in the other one in which a nearly parallel flow is obtained. However, this mean motion is often all that has to be known: no one is interested in the subtle details of the high frequency, small fluctuations of the drag of an airplane, only in the average drag.

On the other hand, extreme values can be important, e.g. the peak thermal loads to be sure the material can withstand them. Similarly, people are interested in the weather forecast in order to find out about tomorrow's weather, but do not care about the average weather. These requirements led to the development of the Large Eddy Simulation (LES) technique, in which a filter is applied to the equations to separate the large enough motions, which can be resolved, from the too small ones, which have to be modeled, in order to solve the most important and energy bearing turbulent motions and get a good idea of the turbulent motion. These techniques are more expensive because a fine grid is required to capture the energy bearing structures and, moreover, the time evolution of these structures has to be computed, while the RANS approach often reduces to a steady problem. Application of the LES technique to an airliner or a car is not foreseen before the middle of the century [77]. Hybrid methods that reduce the computing load by restricting the use of LES techniques to the regions of the flow where they bring significant improvements are presently blooming.

By averaging or filtering the Navier-Stokes equations, a part of the fluid motion is resolved and a part is not. The turbulent, unresolved part appears in the averaged or filtered Navier-Stokes equations as extra terms which represent the mixing of the resolved fluid motion by the unresolved part. The unresolved motion carries momentum, energy or chemical species within the resolved part. The averaging or filtering thus introduces turbulent stresses and heat or species fluxes which have to be modeled. There are six independent components for the turbulent stress tensor and three for the heat or species flux vectors. The system of equations is now unclosed; as there are more unknowns than equations, models are required.

RANS approach

Present status

As pointed out above, turbulent, or Reynolds, stresses and turbulent heat or species fluxes appear in the Reynolds Averaged Navier-Stokes equations. Transport equations for these quantities can be derived from the Navier-Stokes equations. However, the information lost in the averaging process cannot be retrieved, so that these transport equations involve new terms, for which transport equations could be derived, and so on ad infinitum… Modeling is thus required.

In the RANS approach, the modeling of the turbulent stresses and heat or species fluxes heavily relies upon the assumption that the turbulent motion is close to an equilibrium state. Although there is a large range of turbulent scales, with this assumption, the turbulent motion can be characterized by the knowledge of the large, energy bearing scales, the energy distribution in the smaller scales being imposed from the large scales. Therefore, most models only characterize turbulence by two quantities: a turbulent velocity scale and a turbulent length (or equivalently time) scale. The turbulent velocity scale is often deduced from the turbulent kinetic energy, usually labeled k, the transport equation of which is easily derived from the Navier-Stokes equations and the modeling of which is relatively simple.

RANS turbulence models can be sorted into three main groups:
In the first group, Eddy Viscosity Models (EVM) assume an analogy between the mixing of the averaged flow by the turbulent motion and the transport by the Brownian motion of particles within gases to express the turbulent stresses and heat or species fluxes in a way similar to the viscous stresses and heat or species fluxes. They thus introduce an eddy viscosity, thermal conductivity or diffusivity which, unlike its laminar counterpart, is not a property of the fluid but of the flow motion. From dimensional analysis, they are proportional to the product of the turbulent velocity and length scales. This approach is not fully justified: there is no scale separation between the mean and turbulent motions as there is a scale separation between the gas motion and the Brownian motion. Nevertheless, such models are widely used in the industry as they can give fair predictions of simple, sheared flows, such as boundary layers, wakes, mixing layers… which are of large practical importance. The most popular eddy viscosity models were $k-\varepsilon$ models (e.g. [44]) where ε is the turbulent kinetic dissipation rate, i.e. the rate at which turbulent kinetic energy is transformed into heat, and gives a turbulence length scale (see, e.g. [40]). These $k-\varepsilon$ models usually fail to predict boundary layer separation and hence, e.g. ,overestimate the maximum airfoil lift or the compressor performances. They are superseded by the Spalart and Allmaras model [75] and by $k-\omega$ models, mainly the Shear Stress Transfert (SST) model [50], which give improved predictions and are now aeronautic industry workhorses. Again mimicking fluid properties, the thermal conductivity and diffusivity are generally deduced from the eddy viscosity by respectively assuming constant turbulent Prandtl and Schmidt numbers. Although this is nearly the only approach used, it is not fully justified. The standard value for the turbulent Prandtl number (0.9) holds in the main part of the boundary layer, but not very close to the wall, nor in free shear flows.

The second group solves the crudeness of the eddy viscosity assumption which cannot represent correctly all of the components of the turbulent stress tensor and of the heat or species flux vectors. More complex relationships, similar to the ones used in rheology, or derived from tensor representation theorems, can be used to express the turbulent stresses and heat or species fluxes in terms of the turbulence length and velocity scales and of the mean flow gradients. These models are named Non-Linear Eddy Viscosity Models (NLEVM) or Explicit Algebraic Reynolds Stress Models (EARSM) according to the way they are derived. Non-linear representations can also be used to model the heat or species flux vector. These non-linear models provide fair representations of all of the components of the turbulent stress tensor and of the heat or species flux vector and hence better predictions of more complex flows than simple sheared flows, e.g. they can capture some rotation and curvature effects, as shown in Figure 4.

All the above models link the turbulent stresses and heat or species flux to local velocity, temperature and species gradients. However, turbulence does not adjust instantaneously to the mean flow. This leads to the third group of models, solving the transport equations for the turbulent stresses (and for the turbulent heat or species flux) to capture the turbulence memory and non-equilibrium effects. This requires a larger effort as there are six independent components to the turbulent stress tensor and moreover information about the turbulent length scale is still required. The use of Reynolds stress transport equations is an old practice at the academic level (see, e.g. [55]), mainly in pressure-based codes for incompressible flows. Those models seem particularly appropriate to describe flows characterized by separation, rotation and strong curvature effects such as encountered in turbo-machinery [38]. Their introduction in industrial codes, solving averaged Navier-Stokes equations for compressible flows, is a breakthrough of the European research project FLOMANIA [36]. When the thermal problem is considered, three transport equations for the heat flux vector components are to be added, and generally two more transport equations for characteristic scales for the turbulent thermal field, which leads to twelve transport equations. Transport models for the turbulent heat or species fluxes are thus still at the research level.

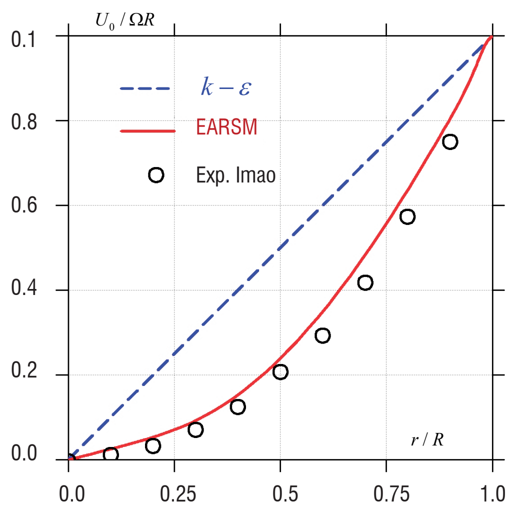

Figure 4 - Mean azimuthal velocity profiles in a rotating pipe: the $k-\varepsilon$ eddy viscosity model cannot capture the rotation effect upon the turbulent motion and predicts a solid body rotation while the EARSM does and gives fair predictions.

Some Onera achievements

As pointed out by the Reno workshop organized by NASA to define the needs in turbulence modeling [67], the prediction of boundary layer separation, which e.g. governs the maximum lift or compressor performance predictions, has been a big challenge for a long time. The idea of imposing mathematical constraints to turbulence models was proposed by Cousteix et al. [24] and extended by Catris and Aupoix [18] to correctly capture the physics of boundary layers close to separation. This finally led to the derivation of the $k-kL$ model by Bézard and Daris [13]. This model has been derived in an eddy viscosity form and in an EARSM form and is one of the models currently used by Dassault Aviation.

Extra transport equations for the thermal scales can be added to get rid of the constant turbulent Prandtl number hypothesis. The above mentioned $k-kL$ model has been complemented with its thermal counterpart as a four equations $k-kL-k_\theta-k_\theta L_\theta$ model. This set of scale equations can be coupled with a simple eddy viscosity/thermal conductivity formulation, thus producing physical variations of the turbulent Prandtl number while keeping the simplicity and robustness of classical two-equation models [12], or with more complex explicit algebraic expressions for improving the Reynolds stress tensor and the heat flux vector representation [31].

Another important problem of turbulent flows comes from the effect of strong deviations from equilibrium caused, for instance, by rapid variations in the mean flow. This aspect is mirrored through the well-known weaknesses of the usual modeled dissipation rate equation. Such complex situations rule out the underlying hypothesis of spectral equilibrium that is implicitly assumed in classical RANS models. For dealing with non-equilibrium situations, new models using several length scales and called multiscale models have been introduced [71] and further developed by split spectrum schemes devised to mimic in an approximate way the change of the spectrum shape. The multiscale concept takes into account some spectral information while staying within the useful framework of the RANS modeling. From a practical point of view, many levels of closure can be considered, but in practice, two spectral slices will be sufficient to describe the effects of non equilibrium distributions. In this context, four-equation multiscale turbulence models based on a split spectrum energy-flux scheme and energy frequency scheme [Masson, 1996] were developed and successfully applied on several basic and more complex 2D and 3D non-equilibrium flows such as shock-boundary layer interaction, transonic channels and an airfoil in stall conditions.

Compressibility can strongly affect the turbulence dynamics. In most aeronautic applications, the turbulent motion remains nearly incompressible, so that the key effect is linked to the mean density variations. Turbulence models are developed for incompressible flows and usually straightforwardly applied to compressible flows. The analysis of scalings in a compressible boundary layer gave the hint that the current practice is not correct and led to the derivation of a general rule to extend any turbulence model to correctly account for density gradients within a boundary layer flow [17].

High speed mixing layers, which are encountered e.g. in scramjets, are among the rare examples of aeronautic flows where the turbulent motion can exhibit a compressible character. The sonic eddy concept [15] states that any turbulent structure must be such that information can circulate within it, i.e. the velocity difference between two points must always be smaller than the speed of sound. This yields a limit on the turbulence length scale, which was first validated and then used to extend in a general way any turbulence model to capture this compressibility effect [6].

As the refraction index is linked to the density, density fluctuations within the flow affect the optical properties of the flow. This has led to the development of aero-optical models for deducing the image blurring from a RANS computation. This requires modeling of both the density fluctuation variance and the way density fluctuations are correlated along the optical path. Models have been derived for boundary layer and mixing layer flows, and validated with respect to LES simulations [83], [7].

Onera has developed a large expertise in flows over rough surfaces, for a wide range of applications such as turbo-machinery or solid propellant rocket nozzles. The standard way to account for wall roughness in industrial codes is the “equivalent sand grain” approach, in which the turbulence model is altered in the wall region to reproduce the drag and heat transfer increases. A general technique to extend any turbulence model to account for wall roughness has been developed and applied to several turbulence models [5], [8]. This technique has also been adapted to account for riblets, small grooves on the wall surfaces like on shark skin, which can equally well lead to a reduction or increase in drag [10].

Of course, most of these models or model improvements are implemented in CEDRE and *elsA*.

Some outlooks

The prediction of the separation point is now fairly well understood. However, the model behavior in the separated region, especially close to the separation and reattachment points, is still an issue as models usually underestimate turbulence in this region. This is one of the topics addressed by the European ATAAC project (http://cfd.mace.manchester.ac.uk/twiki/bin/view/ATAAC/WebHome) in which Onera is involved.

The present industrial trend is to move from eddy viscosity models towards non-linear models and models with transport equations for the Reynolds stresses. A first Reynolds stress model is implemented in *elsA*, others will soon be, and the improvement of the Reynolds stress transport models is one of the present activities of Onera.

It has been shown that classical Reynolds stress transport models, which only use information about the Reynolds stress and the turbulence length scale, are unable to reproduce some flow cases. This is blamed upon the lack of information about the underlying turbulence spatial structures. Cooperation with the University of Cyprus has started on models which account for turbulence structures [9].

For thermal applications, there is a trend to get rid of the constant turbulent Prandtl number assumption, and to move to non-linear representations of the turbulent heat flux vector through explicit algebraic expressions. Onera plans to develop, implement and test improved thermal models, with particular attention to hot exhaust jet applications. The next step of directly transporting the heat flux components is promising but needs further developments before being used industrially.

Approaches to resolving turbulence

Direct Numerical Simulations

New industrial needs in aerodynamics include transient dynamics of separated flows as well as the control of noise so the simulation of unsteady turbulent flows is now required. Consequently, a steady RANS solution would not be what the engineer needs in these kinds of applications. But, remembering that one of the salient features of turbulent flows is their multiscale character, Direct Numerical Simulation can explicitly simulate all of the active scales present in a turbulent flow, since the governing equations are discretized directly and solved numerically. The total number of nodes of such a "modeling free" simulation may scale as $O(\mathrm{Re}_L{}^3)$ where Re_L denotes the Reynolds number based on the integral length scale. If solid walls are present, the near wall structures need to be resolved leading to an even stronger dependence on the Reynolds number. Practical turbulent flows encountered in aeronautics exhibit such a wide range of excited length and time scales (shock waves, boundary and free shear layers,…) at high Reynolds number $(\approx 10^5 - 10^8)$ that DNS becomes inappropriate due to prohibitive cost. In other words, DNS remains an efficient research tool which gives significant insights into turbulence physics but will not be used as a predictive tool for design purposes for at least several decades. This is one of the reason why modeling is necessary prior to solving the Navier Stokes equations.

Large Eddy Simulations

The Large Eddy Simulation approach relies on a decomposition of the field between the large and the small scales of the flow. This approach seeks to directly calculate the largest ones (responsible for turbulence production) while modeling the effects of the smaller-scale eddies. The primary obstacle to practical use of LES on industrial flows which involve wall boundary layers at high Reynolds number remains computing power resources. Indeed, the scales of motion responsible for turbulence production impose severe demands on the grid. In [77], it is proposed that LES of wall turbulence should be considered as a quasi-DNS (QDNS) since the requested resolution for capturing the near wall layer is roughly ten times less expensive than for DNS. The accuracy of DNS/LES for wall-bounded flows has been recently assessed at Onera in various applications including transitional flows [46], [63] and flow control applications [25], [57].

Hybrid methods

Hybrid RANS/LES was invented to alleviate the LES resolution constraints in the near-wall regions. Basically, the objective is to combine the fine-tuned RANS modeling in the attached boundary layers with the accuracy of LES in the separated regions. Hybrid methods can be categorized into two major classes corresponding respectively to "global" and "zonal" hybrid methods (or "weak" and "strong" RANS/LES coupling methods, see Figure 5). We should point out to the reader that some flow situations are characterized by a scale separation between the unsteadiness of the mean field and turbulence. This situation arises when the boundary condition imposes flow unsteadiness (like the flow around a helicopter blade). Subsequently, unsteady statistical approaches like URANS (Unsteady Reynolds Averaged Navier Stokes) might be used. However, many cases such as a landing gear do not have this scale separation.

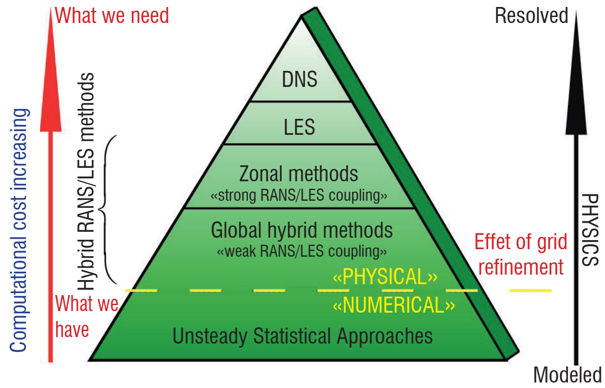

Figure 5 - Classification of unsteady approaches according to levels of modeling and readiness (adapted from Sagaut, Deck and Terracol [68]).

Among hybrid RANS/LES methods, the approach that has probably drawn most attention in the recent time frame is the Detached Eddy Simulation (DES97) which was proposed by Spalart et al. [76] (see also [79]). The idea is to simulate the attached boundary layer in RANS mode whereas the separated flow should be ideally simulated in LES mode. The methods in which the attached boundary layer is modeled in RANS mode can be considered as weak RANS/LES coupling methods since there is no mechanism to transfer the modeled turbulence energy into resolved turbulence energy. These methods introduce a "grey-area" in which the solution in neither pure RANS nor pure LES since the switch from RANS to LES does not imply an instantaneous change in the resolution level. In practice, the eddy viscosity remains continuous across the RANS/LES interface but the rapid decrease of the level of RANS eddy viscosity enables the development of strong instabilities. This family of techniques is well adapted for simulating massively separated flows characterized by a large scale unsteadiness dominating the time-averaged solution (see [68] for further discussion). Two weaknesses in the use of hybrid methods for technical flows have been identified traditionally. The first one concerns a possible delay in the formation of instabilities in mixing layers due to the advection of the upstream RANS eddy viscosity. The second one deals with the treatment of the "grey-area", where the model switches from RANS to LES, and where the velocity fluctuations, the "LES-content", are expected not to be sufficiently developed to compensate for the loss of modeled turbulent stresses ("Model-Stress Depletion" (MSD)). This can lead to unphysical outcomes, like an underestimation of the skin friction which, at worst, can lead to artificial separation denoted as "Grid Induced Separation" (GIS). In order to get rid of this latter drawback, Spalart et al. [78] proposed a modification of the model length scale presented as a Delayed Detached Eddy Simulation (DDES) to delay the switch into the LES mode and to prevent "Model-Stress Depletion". This method, implemented in both CEDRE and els*A* solvers has been successfully used to simulate the buzz in a supersonic inlet [82], side-loads in an over-expanded nozzle flow [30] as well as the reactive flow over a backward facing step [70].

In a different spirit, Deck [26], [27] proposed a Zonal Detached Eddy Simulation (ZDES) approach, in which RANS and DES domains are selected individually. The motivation is to be fully safe from MSD and GIS and to clarify the role of each region. An example of application of this method on a high-lift device is provided in Figure 6. Besides this case, ZDES has been thoroughly validated with experimental data

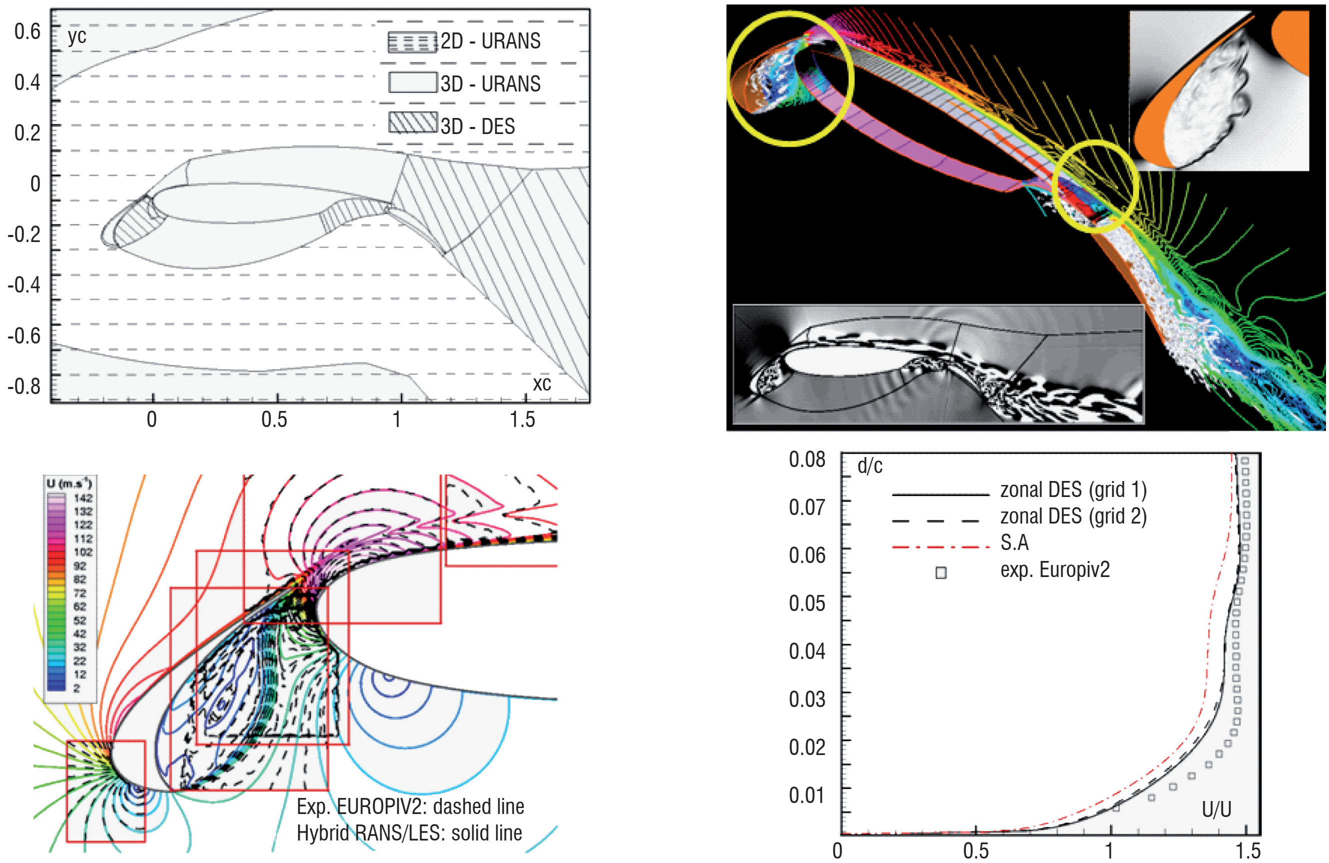

Figure 6 - ZDES of the flow around a three-element airfoil. Instantaneous flow field and comparison of Reynolds averaged data with PIV experimental data (from [27]).

including spectral and second order analysis on a wide range of applications covering both subsonic [28][88] and supersonic [73] base flows and jets [22]. Some other applications in the frame of applications with the *elsA* solver can be found in [62].

Further improvements of this class of hybrid methods, like the Scale Adaptive Simulation (SAS) proposed by Menter et al. [51], are currently being investigated at Onera. Recently, the Partially Integrated Transport Modeling (PITM) method viewed as a continuous approach for hybrid RANS/LES modeling allowing seamless coupling between the RANS and LES regions has been developed in the framework of eddy viscosity models [72] and second moment closures [19], [20], [21], especially for simulating unsteady flows on relatively coarse grids, providing a saving of computing time. From a theoretical point of view, the PITM method finds its basic foundation in the spectral space by considering the Fourier transform of the two-point fluctuating velocity correlation equations in homogeneous turbulence [20]. The extension to non-homogeneous turbulence is developed easily within the approximate framework of the tangent homogeneous space. As a result of the modeling, it is found that the sub-filter stress model relies on transport equations that look formally like the corresponding Reynolds stress transport equations but the coefficients used in the model are no longer constants. They are now some functions of a dimensionless parameter involving the cutoff wave number and of the turbulent length scale built using the total turbulent kinetic energy and the total dissipation rate. The sub-filter stress model has been used for simulating unsteady flows of complex physics encountered in engineering applications such as for instance the injection induced flow viewed in Figure 7. The flow which develops in the channel is subjected to a fluid injection from the lower wall and is bounded by the upper rigid wall. This figure clearly illustrates the three dimensional nature of the flow. These structures are squeezed upwards in the normal direction to the axial flow as previously observed by Apte [2].

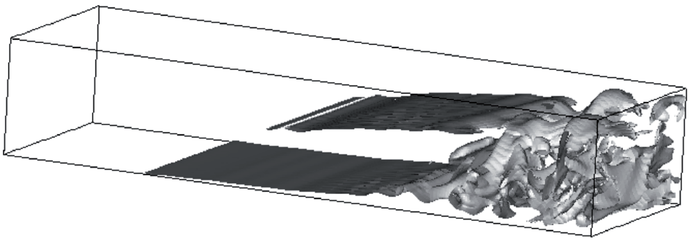

Figure 7 - Isosurface of instantaneous filtered vorticity in the spanwise direction.

Within "weak" RANS/LES coupling methods, boundary layers are treated in RANS mode which can appear to be a limitation in specific cases which are quite sensitive to the Lagrangian history of the upstream or free-stream turbulence. A generic example is provided by a shallow separation bubble on a smooth surface induced by a moderate adverse pressure gradient. The use of predefined "pure" RANS and "pure" LES zones may alleviate this "grey-area" issue. The main problem that arises when dealing with zonal RANS/LES approach originates from the very different spectral content of the solution between these two resolution levels. One of the first attempts to derive a consistent discontinuous coupling between RANS and LES was suggested in [60]. This latter approach is based on the definition of the exchange of information at the RANS/LES interface, which relies on the definition of some interface variables to construct a transfer operator at the interface. In the case where the RANS zone is located downstream a LES domain, Nolin et al. [47], [56] proposed a filtering process to reconstruct an eddy viscosity from the LES field.

The zonal RANS/LES coupling method has been successfully used to simulate the transitional and separated flow around an airfoil near stall [64] (Figure 8). In addition, we should also mention the NLDE (Non-Linear Disturbance Equations) approach extended to the case of a RANS/LES decomposition [42]. Within this perturbation approach, the LES field is broken down as the sum of a mean field (RANS) and a turbulent fluctuation which is computed using modified filtered Navier-Stokes equations (see [80] for a successful application of this method in aeroacoustics).

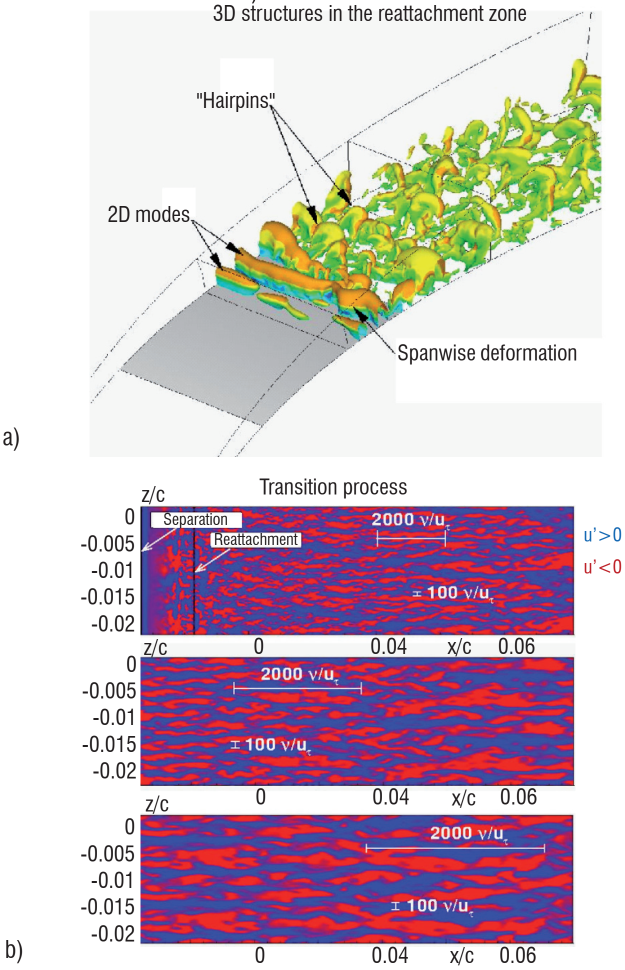

Figure 8 - Transitional and separated flow around an OA209 airfoil near stall.
a) Pressure fluctuation contours $p'=-2,1.10^{-3}$ *Pa*
b) Streamwise velocity fluctuations *u'*

The case where the LES domain is located downstream from a RANS domain is probably the more difficult to deal with since appropriate inflow conditions need to be specified. The generation of inlet conditions for spatially developing turbulent flows remains one of the challenges that must be addressed prior to the application of LES and hybrid RANS/LES to industrial flows [69]. Several techniques including mapping/recycling methods or synthetic turbulence methods have been developed (see [41] for a review). Some recent applications at Onera on synthetic methods can be found in [81], [58], [29], [33] in the frame respectively of NLDE, LES and ZDES.

Some outlooks

In the frame of hybrid RANS/LES methods, it can be concluded that current approaches can handle accurately massively separated flows at high Reynolds numbers for which the location of separation is more or less triggered by the geometry. Conversely, as discussed by Sagaut and Deck [69], one of the next foreseen challenges will be firstly to simulate accurately shallow separation and more generally pressure-gradient-driven separation issues. Such simulations imply the ability to capture accurately the boundary layer dynamics at high Reynolds number and eventually transition. So far, most LES (in a wall-turbulence resolved sense) have concerned low Reynolds number and two-dimensional configurations (the span being considered as a homogeneous direction). The next foreseeable challenge will then concern the ability to handle accurately geometrically complex configurations with validated numerical tools at relevant Reynolds numbers.

Conclusion

This article has given an overview of the variety of modeling approaches presently investigated and developed at Onera, ranging from the solution of averaged equations (e^N approach for transition and RANS approach for turbulent flows) to the model-free solution of the Navier-Stokes equations, through LES and hybrid approaches. Each class of approach has its strengths and deficiencies, in terms of ability to reproduce the physics as well as of computing cost, the more accurate approaches being the more expensive, or even unaffordable. Short and medium term outlooks have been discussed for each type of models. Although industry can currently only deal with averaged approaches for everyday design, developing expertise on each modeling level ensures that Onera is able to respond to all of today's and tomorrow's industrial demands and to improve each modeling level from the knowledge gained from other levels. The large majority of models presented here are available in the CEDRE and/or *elsA* solvers.

Author names appear in alphabetic order.

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Acronyms

2D (Two-Dimensional)
3D (Three-Dimensional)
CF (Cross-Flow)
DES (Detached Eddy Simulation)
DDES (Delayed Detached Eddy Simulation)
DNS (Direct Numerical Simulation)
EARSM (Explicit Algebraic Reynolds Stress Model)
EVM (Eddy Viscosity Model)
GIS (Grid Induced Separation)
LES (Large Eddy Simulation)
MSD (Model-Stress Depletion)
NLDE (Non-Linear Disturbance Equations)
NLEVM (Non-Linear Eddy Viscosity Model)
PITM (Partially Integrated Transport Modeling)
PSE (Parabolized Stability Equations)
QDNS (Quasi Direct Numerical Simulation)
RANS (Reynolds Averaged Navier-Stokes (equations))
SAS (Scale Adaptive Simulation)
TS (Tollmien-Schlichting)
URANS (Unsteady Reynolds Averaged Navier Stokes)
ZDES (Zonal Detached Eddy Simulation)

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