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# The Models of Turbulent Combustion in the CHARME Solver of CEDRE

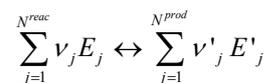
We describe the combustion models implemented in the CHARME solver of the CEDRE code dedicated to the simulation of 3D complex reactive flows. These models give the source terms linked to chemical reactions appearing in the species mass balance equations solved by CHARME. For laminar flows, Arrhenius laws are simply used but, for turbulent flows, different models have been implemented, in order to allow the user to treat diffusion flames, premixed flames or intermediate situations. All of these models are described here. The proposed models are more or less advanced according to the level of accuracy required in regard to the treatment of the turbulence-chemistry interaction. This paper does not deal with more sophisticated models, such as those based on Monte Carlo methods, which require the creation of new solvers for CEDRE.

## Introduction

The CHARME solver is dedicated to the simulation of turbulent reactive gas flows, such as those encountered in gas turbine combustors, rocket engines working with liquid or solid propellants, ramjets, scramjets, etc. The common feature of these flows is that the level of turbulence is kept as high as possible in order to prompt mass and energy transfer, but the type of flame involved in one of these combustors can be different from that in others. In the case of rocket engines using hypergolic fuels the flame can be nothing but a diffusion flame, i.e., a flame in which fuel and oxidant enter the combustion chamber separately and then meet due to transport by diffusion phenomena. On the other hand, in a gas turbine combustor, in order to prevent temperature heterogeneities which may damage turbine blades and promote pollutant emission, organizing the combustion in the shape of premixed flames is increasingly sought. In that case fuel and air are introduced into the combustion chamber by the same injection device (although not always through the same holes of this injection device) and are already mixed or quasi-mixed when they enter the combustion chamber. The combustion propagates in the mixture due to heat conduction from the burnt gases to the fresh ones. Depending on the type of flame, the situation can be very different (Box 1) and can hardly be handled by a unique physical model. Therefore the CHARME solver must contain different models that cover all the situations, from pure diffusion to pure premixed, going through partially premixed flames. Even if most of the flows with combustion are turbulent, laminar flows have to be considered as well.

## Laminar flows

For laminar reactive flows, the reaction rate is given by Arrhenius laws. The CHARME solver gives the user the possibility of introducing any type of chemical kinetics. In that case the user has to specify all the chemical reactions involved in the simulation. A reversible chemical reaction is written:



where  $N^{react}$  (resp.  $N^{prod}$ ) is the number of reactant species (resp. product species) in the reaction, the  $E_j$  (resp.  $E'_j$ ) are the reactant species (resp. product species) and the  $\nu_j$  (resp.  $\nu'_j$ ) are the stoichiometric coefficients of the reactant species (resp. product species). It is possible that species are common to both sides of the reaction when third body effects have to be taken into account.

For each reaction, the user must specify the pre-exponential factor  $A$ , the temperature exponent  $\alpha$  and the activation temperature  $T_A$  for both direct and reverse directions of the reaction. The reaction rate for a given reaction is then written:

$$\dot{\Omega}_r = A_r^{dir} T^{\alpha_r^{dir}} \exp\left(-\frac{T_A^{dir}}{T}\right) \prod_{j=1, N_r^{react}} [c_j^r]^{\nu_j^r} - A_r^{rev} T^{\alpha_r^{rev}} \exp\left(-\frac{T_A^{rev}}{T}\right) \prod_{j=1, N_r^{react}} [c_j^r]^{\nu_j^r} \quad (1)$$

where the subscript  $r$  refers to the reaction, the superscript  $dir$  (resp.  $rev$ ) refers to the direct (resp. reverse) part of the reaction and  $[c]$  (resp.  $[c']$ ) is the concentration of a given reactant (resp. product) of the considered reaction.

Note that to simplify data introduction in the case of hydrogen-methane mixture combustion, the Davidenko kinetics, including 80

reactions and 21 species [1], has been directly coded in the CHARME solver. 16 of these 21 species have to be tracked by transport equations, whereas the last five are deduced from an assumption of reaction partial equilibrium.

### Box 1

In nature, combustion spontaneously appears in the shape of diffusion flame, whereas in industrial or domestic combustion devices it is sought to produce flames which are as premixed as possible.

In a diffusion flame, fuel and oxidant burn very rapidly when they meet and then disappear. Therefore, fuel and oxidant cannot be simultaneously in non-negligible concentrations at the same point of the flow, so that the flame, i. e. the reaction zone, separates the flow into two regions, one containing the fuel and the other containing the oxidant. In the flame, fuel and oxidant have zero or very low concentrations and are replaced by reaction products, i. e. burnt gases. The zero concentration in the reaction zone creates concentration gradients in both fuel and oxidant regions, which result in mass diffusion fluxes that bring fuel and oxidant to the reaction zone, i. e. to the flame. In the left part of Figure B1-1 we can see an example of such a diffusion flame. The combustion zone ends where the less abundant reactant between fuel and oxidant is totally consumed. The length of the flame is given by the proportionality relation:

$$L_f \propto \frac{VS}{D} \quad (1)$$

where  $V$  is the mean velocity of the reactants in the feeding pipes (in m/s),  $S$  is the section of the feeding pipe with the least abundant reactant (in  $m^2$ ) and  $D$  is the mass diffusion coefficient of the reactants into the burnt gases (in  $m^2/s$ ). If the flow is turbulent (as it is most of the time in industrial burners), the turbulent diffusion coefficient  $D_t$  must be considered instead of  $D$  and the flame length is considerably smaller for the same reactant flow rate. A schematic view of a usual diffusion flame is given in the left part of Figure B1-2. Typical concentration profiles obtained across this flame are shown in the upper part of Figure B1-3. Fuel and oxidant mass fraction gradients are in opposite directions and the absolute mass fluxes of fuel and oxidant which meet in the flame are in stoichiometric proportion.



Figure B1-1 - Examples of diffusion flame (left) and premixed flame (right)

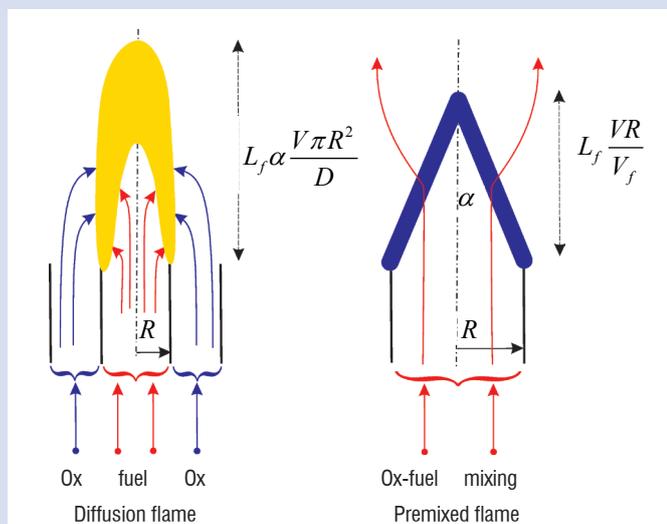


Figure B1-2 - Schematic view of diffusion and premixed flames

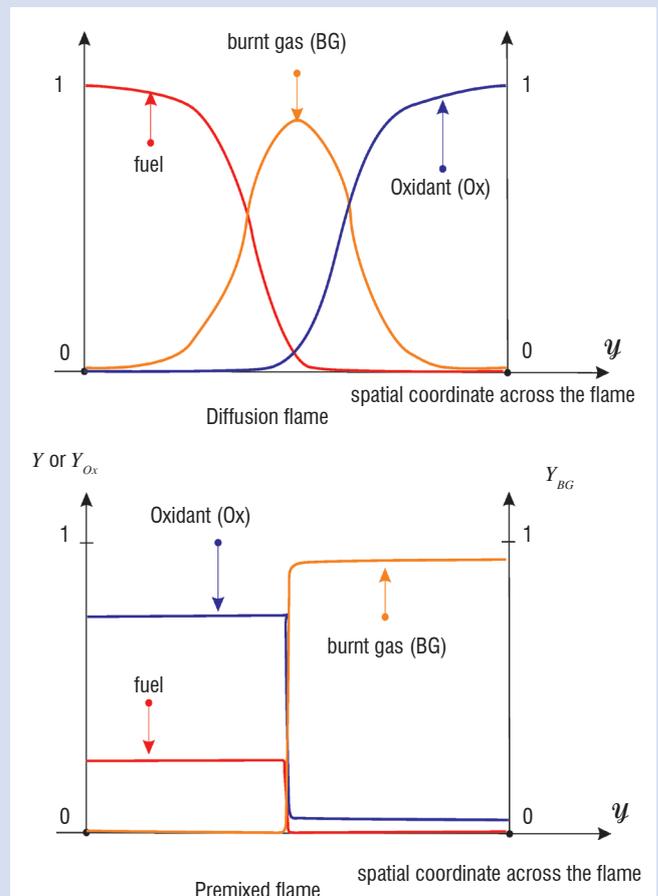


Figure B1-3 - Structure of a diffusion flame (upper part) and a premixed flame (lower part)

In premixed combustion the position of the flame, and hence its length, result from a competition between the flame velocity and the velocity of the mixture. The right part of Figure B1-2 gives the sketch of a Bunsen type premixed flame. The length of the flame is given by:

$$L_f = \frac{R}{\tan \alpha}$$

where  $R$  is the radius of the feeding pipe and  $\alpha$  is the angle defined, as indicated in Figure B1-2. By definition of  $\alpha$ , the flame velocity  $V_f$  is related to the velocity  $V$  of the gas mixture in the feeding pipe by:

$$V_f = V \sin \alpha$$

Assuming for the sake of simplicity that  $\alpha$  is small, one obtains:

$$L_f = \frac{RV}{V_f} \propto \frac{RV}{\sqrt{D_{heat} / \tau_{chem}}} \quad (2)$$

where  $D_{heat}$  is the thermal diffusivity (in  $m^2/s$ ) and  $\tau_{chem}$  is the characteristic time of the chemical kinetics (in s). If the flow is turbulent,  $D_{heat}$  must be replaced by the turbulent heat diffusivity  $D_{t,heat}$  and  $\tau_{chem}$  must be replaced by a chemical time  $\tau_{t,chem}$  obtained by averaging over the thickness of the turbulent flame. The parameter  $D_{heat}$  (resp.  $D_{t,heat}$ ) is linked to the parameter  $D$  (resp.  $D_t$ ) by the well known non-dimensional Lewis number  $D_{heat} / D$  (resp.  $D_{t,heat} / D_t$ ), which is of the order of unity. Note that, in the turbulent case, the thickness of the flame can become of the same order as its length.

The lower part of Figure B1-3 gives the typical concentration profiles in a premixed flame. These profiles are obtained when we follow streamlines crossing the flame front, represented in blue in the right part of Figure B1-2. Unlike the case of a diffusion flame, the concentration profiles of fuel and oxidant have a similar shape and the corresponding gradients are in the same direction; moreover the temperature, the evolution of which is similar to that of the burnt gas mass fraction, maintains its maximum value far from the flame, downstream of the flow.

The differences between Eqs. (1) and (2), and in the concentration profiles, point out that the physics involved for diffusion and premixed flame are different. This difference is obvious when the luminosity of the two types of flames is considered: For the diffusion flames, a yellow colour arises from the light emission by hot soot particles (in the case of CxHy fuel), while premixed flame becomes blue when CxHy decompose into C-C or C-H radicals before burning (Figure B1-1).

## Models for premixed or partially premixed turbulent flames

### Model without the influence of chemistry

When the chemistry is very fast compared to the turbulent mixing, the mean reaction rate can be expressed as a function of the turbulent quantities and mean mass fractions only because turbulent mixing is entirely piloting combustion. For perfectly premixed flows, the well known Eddy-Break-Up model of Spalding [2] leads to a very simple expression of the mean fuel consumption rate  $\tilde{\Omega}$ :

$$\tilde{\Omega} = \frac{C_{EBU}}{\tau_{turb}} \bar{\rho} \tilde{Y} \left( 1 - \frac{\tilde{Y}}{Y_{in}} \right) \quad (2)$$

where  $C_{EBU}$  is a non-dimensional constant (of the order of 5) adjusted by comparison with experiment,  $\tau_{turb}$  is the characteristic turbulent mixing time,  $\bar{\rho}$  is the mean density of the mixture,  $\tilde{Y}$  is the mean fuel mass fraction and  $Y_{in}$  is the initial fuel mass fraction, i.e., the fuel mass fraction before combustion. This formulation can be easily extended to non-premixed flames for which the mixing remains everywhere under stoichiometric conditions [3]:

$$\tilde{\Omega} = \frac{C_{EBU}}{\tau_{turb}} \bar{\rho} \tilde{Y} \left( 1 - \frac{\tilde{Y}}{\tilde{T}_r} \right) \quad (3)$$

where  $\tilde{T}_r$  is a tracking variable, which is a solution of a transport equation with the same boundary conditions as those of  $\tilde{Y}$  but without the source terms of the equations for  $\tilde{Y}$ . In other words  $\tilde{T}_r$  is

the concentration of an inert species injected under the same conditions as the fuel. If the stoichiometry is uniform,  $\tilde{T}_r$  is constant and remains equal to  $Y_{in}$ , so that Eq. (3) is reduced to Eq. (2). A symmetric formulation can be obtained for the mean oxidant consumption rate  $\tilde{\Omega}_{Ox}$  in the much less usual case of a flame that remains totally over-stoichiometric:

$$\tilde{\Omega}_{Ox} = \frac{C_{EBU}}{\tau_{turb}} \bar{\rho} \tilde{Y}_{Ox} \left( 1 - \frac{\tilde{Y}_{Ox}}{\tilde{T}_{rOx}} \right) \quad (4)$$

where the subscript  $Ox$  refers to the oxidant.

The application of Eqs. (3) and (4) only requires the solution of an additional transport equation for an inert species, but the extension of the model to mixing fluctuations across stoichiometry is less simple. In that case, the Shvab-Zeldovich variable  $\varphi$  must be introduced. By definition, this variable  $\varphi$  is equal to  $\beta Y - Y_{Ox}$ , where  $\beta$  is the ratio of oxidant mass to fuel mass under stoichiometric conditions. Source terms of the transport equations of  $\beta Y$  and  $-Y_{Ox}$  compensate each other so that  $\varphi$  is really an inert species. This makes it possible to presume its Probability Density Function (PDF)  $P(\varphi)$  from its mean value  $\tilde{\varphi}$  and its fluctuation  $\tilde{\varphi}''^2$  because in that case the transport equation of  $\tilde{\varphi}''^2$  does not include source terms resulting from coupling with chemistry. The location of the gaseous particles in the phase plane  $(Y, \varphi)$  is indicated in Figure 1. In this section we assume that combustion leads to the total consumption of the less abundant reactant, i.e. the product gases are located on the red line of Figure 1. Using this assumption, the extended expression of  $\tilde{\Omega}$  can be easily obtained (see [3]):

$$\tilde{\Omega} = \frac{C_{EBU}}{\tau_{turb}} \bar{\rho} \times \left[ \tilde{Y} \int_{\min(0, \varphi_{\min})}^0 \alpha P(\varphi) d\varphi + \frac{\tilde{Y}_{Ox}}{\beta} \int_0^{\max(0, \varphi_{\max})} \alpha P(\varphi) d\varphi \right] \quad (5)$$

where  $\alpha$  is given by

$$\alpha = \frac{Y_{\min} + (\tilde{\varphi} - \varphi_{\min}) \frac{Y_{\max} - Y_{\min}}{\varphi_{\max} - \varphi_{\min}} - \tilde{Y}}{Y_{\min} + (\tilde{\varphi} - \varphi_{\min}) \frac{Y_{\max} - Y_{\min}}{\varphi_{\max} - \varphi_{\min}} - \int_0^{\max(0, \varphi_{\max})} \frac{\varphi}{\beta} P(\varphi) d\varphi} \quad (6)$$

In these equations the subscript min (resp. max) refers to the minimum (resp. maximum) value taken by  $Y$  or  $\varphi$  in the flow inlets. This means that the corresponding quantities are perfectly known as a part of the boundary conditions.

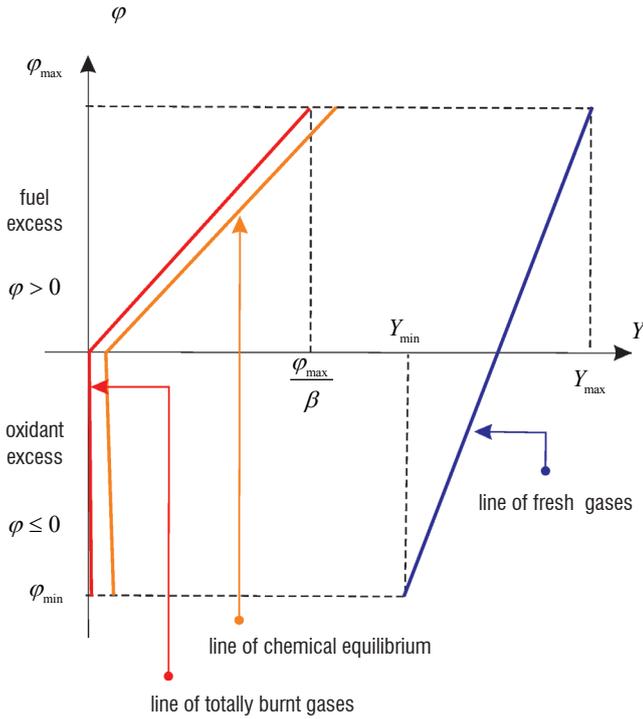


Figure 1 - Location of the gaseous particles in the  $(Y, \varphi)$  plane

Note that Eqs. (5) and (6) reduce to Eq. (3) when the mixing remains under stoichiometric conditions everywhere: Thus, the second integral of Eq. (5) vanishes since  $\varphi_{\max}$  is less than zero; in the same way the integral in Eq. (6) also vanishes and it is not difficult to establish that the quantity:

$$Y_{\min} + (\tilde{\varphi} - \varphi_{\min}) \frac{Y_{\max} - Y_{\min}}{\varphi_{\max} - \varphi_{\min}}$$

is identical to the quantity  $\tilde{T}_r$  of Eq. (3); in addition, if we take into

account that  $\int_{\min(0, \varphi_{\min})}^0 P(\varphi) d\varphi$  is equal to 1 by definition of a PDF,

the proof is complete.

Eqs. (5) and (6) make the basis of the so called CRAMER combustion model for premixed and partially premixed flames, which was implemented in the CHARME solver. This model assumes that the turbulent mixing of fresh and hot gases is slower than the chemical kinetics and thus entirely drives the mean reaction rate. In the case

of a variable fuel-air mixture ratio, it requires the resolution of two additional transport equations for  $\tilde{\varphi}$  and  $\tilde{\varphi}^2$  and the evaluation of the PDF of the inert variable  $\varphi$ . In the case of a constant mixture ratio, no additional transport equations are needed and the model reduces to the introduction of a source term in the fuel mass transport equation, calculated according to Eq. (4).

### Model taking into account chemical equilibrium data

The CRAMER model does not take into account the chemical equilibrium data in the sense that combustion leads to complete consumption of the fuel (resp. oxidant) in under-stoichiometric (resp. over-stoichiometric) conditions. In reality, at the end of the reaction the obtained temperature is lower than the temperature corresponding to a complete conversion of the fresh gases into CO<sub>2</sub> and H<sub>2</sub>O. The energy really obtained corresponds to a non complete conversion of the fuel (or oxidant in the case of fuel excess), i.e. a conversion which would stop at a non zero limiting value of the mass fraction of the fuel (or oxidant in the case of fuel excess). The corresponding location of the gaseous particles is the beige line in Figure 1. If we perfectly know the shape of this line, it is possible to extend Eqs. (5) and (6) as follows:

$$\tilde{\Omega} = \frac{C_{EBU}}{\tau_{turb}} \bar{\rho} \times \left[ \int_{\min(0, \varphi_{\min})}^0 (\tilde{Y} - Y_{eq}(\varphi)) \alpha P(\varphi) d\varphi + \frac{1}{\beta} \int_0^{\max(0, \varphi_{\max})} (\tilde{Y}_{Ox} - Y_{eqOx}(\varphi)) \alpha P(\varphi) d\varphi \right] \quad (7)$$

with  $\alpha$  given by

$$\alpha = \frac{A - \tilde{Y}}{A - B} \quad (8)$$

$$A = Y_{\min} + (\tilde{\varphi} - \varphi_{\min}) \frac{Y_{\max} - Y_{\min}}{\varphi_{\max} - \varphi_{\min}}$$

$$B = \int_0^{\max(0, \varphi_{\max})} \frac{\varphi + Y_{eqOx}(\varphi)}{\beta} P(\varphi) d\varphi - \int_{\min(0, \varphi_{\min})}^0 Y_{eq}(\varphi) P(\varphi) d\varphi$$

where the subscript eq refers to the limiting value imposed by chemical equilibrium. The calculation of  $\alpha$  and then  $\tilde{\Omega}$  must be achieved in each cell at each time step. It is not CPU time consuming, because the PDF  $P(\varphi)$  is modelled in the shape of a sum of delta and constant value functions, which leads to a very simple calculation of the integrals involving the variable  $\varphi$ . Eqs. (7) and (8) are the basis of the so called CLE ("Combustion Limitée par l'Equilibre") model [4], which was implemented in the CHARME solver. The preliminary calculation of the functions  $Y_{lim}(\varphi)$  and  $Y_{limOx}(\varphi)$  corresponding to the chemical equilibrium at the pressure of the reactive flow is required to use this model.

### Model taking into account chemical kinetics

The combustion regime of a reactive flow can be defined by use of the Borghi-Peters Diagram [5,6]. The combustion regime obtained in many industrial applications corresponds to chemical reactions taking place in wrinkled thickened flames. A straightforward idea to take into account the interaction between turbulence and combustion is

therefore to associate presumed PDF models for relevant combustion parameters with tabulated results of one dimensional premixed flame calculations. The relevant parameters are the progress variable  $c$ , which is monotonously varying from 0 in the fresh gases, to 1 in the burnt gases and, if the fuel-air mixture ratio is not uniform, the mixing variable  $z$ , which is monotonously varying from 0 on the oxidant side to 1 on the fuel side. There is obviously a linear relation between  $z$  and the previously introduced variables  $\varphi$  and  $T_r$ .

Some preliminary calculations of 1D premixed flames with detailed chemical kinetics give the gaseous mixing composition for any value of the progress variable  $c$ . If the fuel-air mixture ratio is uniform, only one 1D premixed flame calculation has to be achieved. If not, 1D premixed flame calculations have to be done for a set of  $z$  values covering the whole range of mixture ratio variation. The results are gathered in a table from which information on the mixing composition is extracted during the calculation of the reactive flow. If the fuel air mixture ratio is uniform, the variable  $c$  is the only entry of the table. If not, the table has the two entries  $c$  and  $z$ , the latter being used to select the premixed flame with the same fuel-air mixture ratio as that at the point where the composition is sought. In the calculation of the reactive flow, transport equations are solved only for the main species such as fuel, O<sub>2</sub>, CO, CO<sub>2</sub>, H<sub>2</sub>O. The mean source term for these species is obtained by convolution of the source term given by the table with the PDF of  $c$  and  $z$ . Thus:

$$\tilde{\Omega} = \iint_{c,z} \dot{\Omega}_{tab}(c,z) P(c,z) dc dz \quad (9)$$

The detailed composition of the flow is obtained from the table as well; the mass fraction  $Y^{minor}$  of the minor species is given by:

$$\tilde{Y}^{minor} = \iint_{c,z} Y_{tab}^{minor}(c,z) P(c,z) dc dz \quad (10)$$

This kind of model was introduced a long time ago (see for example [7]) and has been widely used since [8].

The determination of the PDF  $P(c,z)$  is not simple. It relies on the assumption that the correlation between  $c$  and  $z$  can be neglected and that their PDFs can be modelled from their first ( $\tilde{c}$  and  $\tilde{z}$ ) and second ( $\tilde{c}^2$  and  $\tilde{z}^2$ ) moments only. Additional transport equations are solved for these quantities. The closure of the first moment equations is simple, since  $c$  and  $z$  can be expressed as a function of the major species mass fractions. Thus,  $c$  can be defined as:

$$c = \frac{Y_{CO} + Y_{CO_2}}{Y_{CO}^{eq} + Y_{CO_2}^{eq}} \quad (11)$$

where the superscript eq refers to chemical equilibrium. The closure of the second moment equations is more complicated, especially for  $\tilde{c}^2$ , because its equation contains some correlations between source terms and concentrations. The details on the solution of these equations, which have been introduced in the CHARME solver of CEDRE, can be found in [9, 10].

### Model for Large Eddy Simulation (LES)

To save computation time, the reaction rate in LES type simulations is generally given by a global chemical kinetics adjusted to give the appropriate flame speed in one dimensional calculations. In the 3D

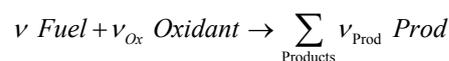
LES calculations, the flame propagation must be correctly reproduced, though the calculation grid is not refined enough to give a good description of the flame front most of the time. To comply with this constraint, the mass diffusion coefficient and the chemical kinetics parameters are modified in the LES to artificially thicken the flame front, keeping the flame speed unchanged. On the other hand, thick flames are less stretched and folded by turbulent structures than ordinary flames, so that the consumption rate of reactants is lowered. To offset this non physical effect, the reaction rate initially given by the global kinetics is multiplied by a so called Efficiency Function accounting for the real stretch and folding of the flame. All these ingredients make up the TFLES (Thickened Flame for LES) model [11], which has been introduced in the CHARME solver.

### Models for diffusion flames

As explained in box 1, the rate of reactant consumption in diffusion flames depends much more on the intensity of the mass diffusion fluxes than on the chemical kinetics<sup>1</sup>. Therefore, models of turbulent combustion for diffusion flames are mainly based on turbulence parameters and often do not involve chemical data. The simplest model for diffusion flames introduced in the CHARME solver is the popular Magnussen model [12]. According to this model, the mean reaction rate was initially given by:

$$\tilde{\Omega} = \frac{C_{Mag}}{\tau_{turb}} \min\left(\frac{Y}{\nu M}, \frac{Y_{Ox}}{\nu_{Ox} M_{Ox}}\right) \quad (12)$$

where  $C_{Mag}$  is a non-dimensional constant to be adjusted by comparison with reference experiments,  $\nu$  is the stoichiometric coefficient in the global reaction:



$M$  is the molar mass and the index refers to the oxidant. Quantities without index relate to the fuel. This model has been introduced into the CHARME solver in a slightly modified form, to take into account the influence of an ignition temperature. Thus, the mean reaction rate is written

$$\tilde{\Omega} = \frac{C_{Mag}}{\tau_{turb}} H(T - T_{ig}) \min\left(\frac{Y}{\nu M}, \frac{Y_{Ox}}{\nu_{Ox} M_{Ox}}\right) \quad (13)$$

where  $H$  is the Heaviside function ( $H(x) = 1$  if  $x \geq 0$ ,  $H(x) = 0$  otherwise),  $T$  is the gas temperature and the index  $ig$  refers to a threshold temperature. The reaction cannot start as long as the fuel and oxidant have not met and the gas temperature remains lower than  $T_{ig}$ .

Similarly to premixed flames, the notion of laminar flames can be used to build models for turbulent diffusion flame. Indeed, a turbulent diffusion flame can be considered as a laminar diffusion flame which is folded, stretched and sometimes extinguished by the turbulence. This idea gave birth to the coherent flame model [13], which states that the mean reaction rate of a reactant (fuel or oxidant) is the product of its consumption rate per flame surface unit multiplied by the flame surface per volume unit. Thus, the reaction rate of the reactant with index  $i$  is written:

$$\tilde{\Omega}_i = V_i \Sigma \quad (14)$$

<sup>1</sup> Chemical kinetics becomes important in diffusion flames if we want to calculate the flame stabilization process which takes place in small recirculation zones or in points where the flow velocity is low. Triple flames are often the key to this stabilization process.

where  $V_i$  is the consumption rate of the species  $i$  per flame surface unit and  $\Sigma$  is the flame surface per volume unit.

In its first version, the sub-model giving  $V_i$  accounts for the effects of mass diffusion and flame stretch but not for those of chemical kinetics. In the CHARME solver, the fuel consumption rate per flame surface  $V$  is therefore calculated as follows:

$$V = Y(\infty) \sqrt{\frac{D \varepsilon_{sr}}{2\pi}} \frac{\Phi + 1}{\Phi} e^{-W^2} \quad \text{with } W = \text{erf}^{-1} \left( \frac{\Phi - 1}{\Phi + 1} \right) \quad (15)$$

where  $Y(\infty)$  is the fuel mass fraction far from the flame, i. e. the fuel mass fraction in the fuel inlet section,  $D$  is the mass diffusion coefficient of the fuel in the gas mixing,  $\varepsilon_{sr}$  is the stretch rate of the flame, and  $\Phi$  is the stoichiometric ratio calculated from the fuel and oxidant mass fraction in the inlet sections ( $\Phi=1$  at stoichiometry).

The quantity  $\Sigma$  is obtained by solving the following transport equation:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\tilde{v} \Sigma) = \nabla \cdot \left( \frac{v_t}{Sc_\Sigma} \nabla \Sigma \right) + \varepsilon_{sr} \Sigma - \left( \frac{V}{y} + \frac{V_{ox}}{y_{ox}} \right) \Sigma^2 \quad (16)$$

where  $\tilde{v}$  is the mean velocity of the flow,  $v_t$  is the kinematic turbulent viscosity,  $Sc_\Sigma$  is the non-dimensional Schmidt number for the quantity  $\Sigma$  (of the order of unity) and  $y$  (resp.  $y_{ox}$ ) is the mass fraction of the fuel (resp. oxidant). The two terms on the left hand side and the first term on the right hand side are the usual unsteady, convection and diffusion terms of a transport equation. The second and third terms on the right hand side stand for the creation of  $\Sigma$  by flame stretch and its consumption by the vanishing of one reactant (fuel or oxidant), respectively.

Compared to the Magnussen model, this model requires the solution of one additional transport equation for  $\Sigma$ , but describes the physics involved in a turbulent diffusion flame more accurately. Moreover, the sub-model for  $V_i$  can be easily adapted to take into account the effect of the chemical kinetics. A table containing the value of  $V_i$  for each value of  $\varepsilon_{sr}$  could be created from preliminary calculations of stretched laminar diffusion flames, performed with detailed chemical kinetics ■

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## Acronyms:

CEDRE (Calcul d'Écoulements Diphasiques Réactifs pour l'Énergétique)  
 CLE (Combustion Limitée par l'Équilibre)  
 CRAMER (Combustion RAPide pour Mélanges Évolutifs en Richesse)  
 DEFA (Département d'Énergétique Fondamentale et Appliquée)  
 LES (Large Eddy Simulation)  
 PDF (Probability Density Function)  
 TFLES (Thickened Flame model for LES)



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